(26) Seat NO: ______

No of printed pages: 2

Sardar Patel University

Mathematics

M.Sc. Semester I

Monday, 17 October 2016

10.00 a.m. to 1.00 p.m.

PS01CMTH01 - Complex Analysis I

Maximum Marks: 70

Q.1 Fill in the blanks. [8]

(1) Suppose C is a positively oriented simple closed contour and 0 is interior to C. Then the value of $\frac{1}{2\pi i}\int_C \frac{dz}{z^2}$ equals ____

(a) 0 (b) 1 (c) 2 (d) None of these

(2) Let C be the positively oriented circle $|z| = 1$. Then the value of $\int_C e^{1/z}dz$ equals ____

(a) 0 (b) 1 (c) $2\pi i$ (d) None of these

(3) The maximum value of $|\sin z|$ in the rectangular region $0 \leq x \leq \pi$, $0 \leq y \leq 1$ is found at ____

(a) $(\frac{\pi}{2}, 1)$ (b) $(\pi, 1)$ (c) $(0, 1)$ (d) None of these

(4) Suppose u is a harmonic on a domain D. Then the value of $\frac{\partial^2 u}{\partial z \partial \bar{z}} =$ ____

(a) 1 (b) 0 (c) -1 (d) None of these

(5) The residue of a function at a removable singularity is ____

(a) 0 (b) 1 (c) $2\pi i$ (d) None of these

(6) Suppose z is either real or purely imaginary. Then ____

(a) $z^2 = \bar{z}$ (b) $(\bar{z})^2 = z$ (c) $(\bar{z})^2 = z^2$ (d) None of these

(7) The equation $|z - 1| = |z + i|$ determines a ____

(a) circle (b) ellipse (c) pair of lines (d) none of these

(8) The value of $\int_{|z|=2} \frac{\cosh z}{z^4} dz$ is ____

(a) $i\pi$ (b) $2i\pi$ (c) $\frac{1}{3}i\pi$ (d) none of these

Q.2 Attempt any *Seven*. [14]

(a) Find the value of $\sqrt{i} + \sqrt{-i}$.

(b) Deduce Cauchy-Goursat Theorem from Cauchy Integral Formula.

(c) Prove Gauss Mean Value Theorem.

(P.T.O.)

(d) If $\lim_{z\to 0} \frac{1}{f(1/z)} = 0$, then show that $\lim_{z\to\infty} f(z) = \infty$.

(e) Find the inverse of a bilinear transformation.

(f) Find the value of $\int_{|z|=1} ze^{z} dz$.

(g) Show that limit of a function is unique.

(h) If u is a harmonic conjugate of itself, then show that u is constant.

(i) Show that 0 is a pole of order 3 of $\frac{\sinh z}{z^4}$.

Q.3

(a) Find out the condition precisely for which $\sin(i\bar{z}) = \overline{\sin(iz)}$. [6]

(b) Find out the complex number $(1 + \sin\alpha + i\cos\alpha)^n/(1 + \sin\alpha - i\cos\alpha)^n$. [6]

OR

(b) Find out the n^{th} roots of unity and hence find out the 4^{th} roots and give their geometrical configuration. [6]

Q.4

(c) What are the necessary conditions for the existence of the derivative of a function at a point? Are they sufficient? Justify your answer. [6]

(d) Let f be analytic on a domain D. If $f'(z) = 0$ for all $z \in D$, then show that f is a constant map. [6]

OR

(d) Suppose $f(z) = u + iv$ on a domain D. Show that f is continuous on D if and only if u and v are continuous on D. [6]

Q.5

(e) State and prove Cauchy's Integral Formula and explain its meaning. [6]

(f) Deduce Fundamental Theorem of Algebra from a well known theorem. State the result used here. [6]

OR

(f) Let f be a analytic on a domain $|z - z_0| < \epsilon$. If $|f(z)| \leq |f(z_0)|$ for all z in the domain, then show that f is constant. [6]

Q.6

(g) State and prove Taylor's Theorem. [6]

(h) State and prove Cauchy's Residue Theorem. Can Cauchy-Goursat Theorem be deduced from this result? Justify your answer. [6]

OR

(h) State Laurent's Theorem and find out the Laurent series of $\frac{1}{(z-1)(2-z)}$ in $1 < |z| < 2$ and $|z| > 2$. [6]