[79] [79]

SEAT No.________ NO. OF PRINTED PAGES: 2

SARDAR PATEL UNIVERSITY
M.Sc. (Semester-I) Examination
March-April-2019
Monday01/04/2019
Time: 10:00AM to 01:00 PM
Subject: Mathematics
Course No.PS01EMTH22 (Mathematical Classical Mechanics)

Note: (1) All questions (including multiple choice questions) are to be answered in the answer book only.
(2) Numbers to the right indicate full marks of the respective question.

Q-1 Choose most appropriate answerfrom the options given. (08)

(1) If V_1 and V_2 are potentials corresponding to a conservative force $\bar{F}$ then ______
(a) $V_1 = V_2$ (b) $V_1 = V_2 + constant$ (c) $V_1 V_2 = 0$ (d) nothing can be said

(2) For the motion of a particle inside a circle constraints are ...
(a)holonomicand rheonomic (b) non-holonomic and rheonomic
(c) holonomic and scleronomic (d) non-holonomic and scleronomic

(3) Degrees of freedom of rigid body is
(a) 1 (b) 3 (c) 2 (d) 6

(4) Which one of the following is correct?
(a)Lagrangian is unique. (b) Potential energy is unique.
(c) Hamiltonian is unique. (d) None of these.

(5) The condition for extremum of $\int_{x_1}^{x_2} f(y, \dot{y}, x)\, dx$ is
(a) f is constant (b) $\frac{d}{dx}\left(\frac{\partial f}{\partial y}\right) - \frac{\partial f}{\partial \dot{y}}=0$
(c) $\frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}}\right) - \frac{\partial f}{\partial y}=0$ (d) $\frac{d}{dy}\left(\frac{\partial f}{\partial x}\right) - \frac{\partial f}{\partial \dot{y}}=0$

(6) If $\frac{\partial L}{\partial t} = 0$ then
(a) Lagrangian is conserved (b) Hamiltonian is conserved
(c) energy function is conserved (d) nothing is conserved

(7) Pick up the incorrect statement:
(a) Determinant of a symplectic matrix is zero.
(b) Identity matrix is symplectic.
(c) Product of two symplectic matrices is symplectic.
(d) A symplectic matrix is non-singular.

(8) For generalized momenta $p_1$and p_2, $[p_1, p_2]$ =______.
(a) 0 (b) 1 (c) $p_1 p_2$ (d) -1

Q-2 Answer any **Seven.** (14)

(1) State constraints for the motion of a particle on a sphere.
(2) State Lagrange's equations of motion in case of a velocity dependent potential.
(3) State the expression of the action integral.
(4) What is the curve for a minimum surface of revolution?
(5) State Hamilton's modified principle.
(6) What is a Legendre transformation?
(7) State the symplectic condition for a canonical transformation.
(8) State expression for Poisson bracket in matrix form.
(9) Define Lagrange bracket.

(P.T.O.)

Q-3

(a) State Lagrange's equations of motion in general form and derive Lagrange's equations in case of a conservative force. (06)

(b) Explain the meaning of constraints. Explain the meaning of a scleronomic constraint giving an example. (06)

OR

(b) Giving all details obtain Lagrangian for a simple harmonic oscillator.

Q-4

(a) Discuss conservation of linear momentum using Lagrangian formalism. (06)

(b) Using calculus of variations obtain the curve of the minimum surface area of revolution about y-axis. (06)

OR

(b) Lagrangian for a system is given by $L=\frac{m}{2}(\dot{r}^2+r^2\dot{\theta}^2)-\frac{k}{r}$. Obtain energy function. Is it conserved? Justify your answer.

Q-5

(a) State Hamilton's equations of motion in matrix form and verify it for two degrees of freedom. (06)

(b) The Lagrangian L of a system is given as (06)
$L=\frac{m}{2}\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)+\frac{wm}{2}(x\dot{y}-y\dot{x})$. Obtain Hamiltonian's equations of motion.

OR

(b) Lagrangian for a system of three degrees of freedom is given by
$L=\frac{I_1}{2}\left(\dot{\theta}^2+\dot{\phi}^2\sin^2\theta\right)+\frac{I_3}{2}\left(\dot{\psi}+\dot{\phi}\cos\theta\right)^2-Mgl\cos\theta$. Obtain Routhian.

Q-6

(a) In usual notations show that (06)
$u[t]=u_0+t[u,H]_0+\frac{t^2}{2!}\left[[u,H],H\right]_0+\cdots$

(b) What is meant by fundamental Poisson brackets? Obtain all possible fundamental Poisson brackets. (06)

OR

(b) Show that the transformation,
$Q=\log\left(1+\sqrt{q}\cos p\right), P=2\sqrt{q}\left(1+\sqrt{q}\cos p\right)\sin p$,
is canonical.

****X****