SEAT NO.________ No. of printed pages 2

[78]

Sardar Patel University

M.Sc.(Sem-I), PS01EMTH21, Graph Theory I;

Monday, 1st April, 2019;10.00 a.m. to 01.00 p.m.

Maximum Marks : 70

Note: (i) Notations and terminologies are standard; (ii) Figures to the right indicate marks.

Q.1 Choose the most appropriate option in the following questions. [08]

1. For $G = C_7$, if $D = diam(G)$ and $R = rad(G)$, then

   (a) $D = R$ (b) $D = 2R$ (c) $D = 3R$ (d) None of these

2. If G is complete symmetric digraph with n vertices, then $|E(G)| =$

   (a) n (b) $n(n-1)$ (c) n^2 (d) $\frac{n(n-1)}{2}$

3. Let T be a spanning out-tree with root R. Then

   (a) $d^+(R) > 0$ (b) $d^+(R) = 0$ (c) $d^+(R) < 0$ (d) None of these

4. If G is a simple digraph with vertices $\{v_1, v_2, v_3, \ldots, v_n\}$ & e edges, then $\sum_{i=1}^{n} d^+(v_i) =$

   (a) ne (b) e^2 (c) $2e$ (d) e

5. The coefficient c_5 in Chromatic polynomial of K_5 is

   (a) 0 (b) 1 (c) 5 (d) 5!

6. Which of the following graph is not Hamiltonian?

   (a) K_n (b) P_n (c) C_n (d) None of these

7. If $G = P_{2021}$, then

   (a) $\alpha(G) = \beta(G)$ (c) $\alpha'(G) = \beta(G)$

   (b) $\alpha'(G) = \beta'(G)$ (d) None of these

8. If $G = C_7$ and M is a maximal matching in G, then $|M| =$

   (a) 2 (b) 3 (c) 4 (d) None of these

Q.2 Attempt any seven. [14]

1. Define Euler graph.
2. Write any four properties of tree.
3. Define spanning in-tree.
4. Define Incidence matrix of digraph.
5. Find Chromatic number of C_7.
6. What is four color problem?
7. If G is a tree, then show that $\chi(G) = 2$.
8. Write Hall's matching condition.

(P.T.O.)

9. Define perfect matching.

Q.3

(a) Discuss in detail the Teleprinter problem for $r = 4$. [06]

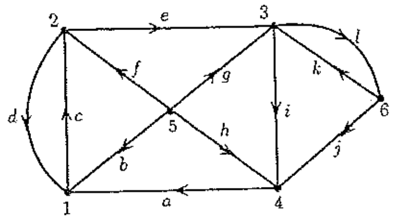
(b) Define the following with an example (1) Eccentricity of vertex, (2) Center of graph and (3) Radius of graph. [06]

OR

(b) Define the following with an example (1) Balance digraph, (2) Regular digraph and (3) Euler digraph. [06]

Q.4

(a) Find fundamental circuit matrix of the following diagraph. [06]



(b) Show that the determinant of every square sub matrix of the incidence matrix A of a digraph is $1, -1$ or 0. [06]

OR

(b) Let X denote the adjacency matrix of digraph $G = (V, E)$. Show that $(i, j)^{th}$ entry of $X^r (r \in \mathbb{N})$ is the total number of directed edge sequence from v_i to v_j of length r. [06]

Q.5

(a) Let G be a connected graph. Then show that G is 2-chromatic if and only if G does not contain a cycle of odd length. [06]

(b) Find Chromatic polynomial of $K_{2,2}$. [06]

OR

(b) Suppose G is Hamiltonian graph. Show that for any non-empty $S \subset V(G)$, $c(G-S) \leq |S|$. [06]

Q.6

(a) State and prove Min-max Theorem. [06]

(b) Let $G = K_{3,4}$. Then find a [06]
(1) minimal vertex cover of G and $\beta(G)$,
(2) minimal edge cover of G and $\beta'(G)$ and
(3) maximal matching in G and $\alpha'(G)$

OR

(b) For $k > 0$, show that every $k-$ regular bipartite graph has a perfect matching. [06]

—X—