[77] [77] SEAT No._____ NO. OF PRINTED PAGES: 2

SARDAR PATEL UNIVERSITY
M.Sc. (Semester-I) Examination
March-April-2019
Monday 01/04/2019
Time: 10:00 AM to 01:00 PM
Subject: Mathematics
Course No.PS01EMTH02 (Mathematical Classical Mechanics)

Note: (1) All questions (including multiple choice questions) are to be answered in the answer book only.
(2) Numbers to the right indicate full marks of the respective question.

Q-1 Choose most appropriate answer from the options given. (08)

(1) For a conservative system _____ is conserved
(a) total energy (b) total angular momentum
(c) total linear momentum (d) nothing

(2) For the motion of a particle on a cylinder constraints are
(a) holonomic and rheonomic (b) non-holonomic and rheonomic
(c) holonomic and scleronomic (d) non-holonomic and scleronomic

(3) Degrees of freedom for a particle moving in the space is
(a) 1 (b) 3 (c) 2 (d) 6

(4) Which one of the following is correct?
(a) Lagrangian is unique (b) Potential energy is unique.
(c) Hamiltonian is unique. (d) None of these.

(5) The condition for extremum of $\int_{x_1}^{x_2} f(y, \dot{y}, x)\, dx$ is
(a) f is constant (b) $\frac{d}{dx}\left(\frac{\partial f}{\partial y}\right) - \frac{\partial f}{\partial \dot{y}} = 0$
(c) $\frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}}\right) - \frac{\partial f}{\partial y} = 0$ (d) $\frac{d}{dy}\left(\frac{\partial f}{\partial x}\right) - \frac{\partial f}{\partial \dot{y}} = 0$

(6) If $\frac{\partial H}{\partial t} = 0$ then
(a) Lagrangian is conserved (b) Hamiltonian is conserved
(c) all momenta are conserved (d) nothing is conserved

(7) Pick up the incorrect statement:
(a) Determinant of a symplectic matrix is greater than one
(b) Identity matrix is symplectic.
(c) Product of two symplectic matrices is symplectic.
(d) A symplectic matrix is non-singular.

(8) For generalized coordinates q_1 and q_2, $[q_1, q_2] =$_____.
(a) 0 (b) 1 (c) q_1q_2 (d) -1

Q-2 Answer any **Seven.** (14)

(1) State constraints for the motion of a particle on an ellipse.
(2) State Lagrange's equations of motion in case of presence of a frictional form.
(3) State condition for extremum of the action integral..
(4) What is the curve for a brachistochrone?
(5) State Hamilton's modified principle.
(6) What is a Legendre transformation?
(7) State transformation equations for a generating function of type $F_2 = q_iP_i$.
(8) State Jacobi's identity for Poisson brackets.
(9) Show that Poisson brackets are linear in first argument..

(P.T.O)

Q-3

(a) State D' Alembert's principle and derive Lagrange's equations from it. (06)

(b) Explain the meaning of constraints. Explain the meaning of a non-holonomic constraint giving an example. (06)

OR

(b) Giving all details obtain Lagrangian for a simple pendulum.

Q-4

(a) Derive Euler-Lagrange equations in the form $\frac{\partial f}{\partial y_i} - \frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}_i}\right) = 0$. (06)

(b) Using calculus of variations obtain the curve of the minimum surface area of revolution about y-axis. (06)

OR

(b) Discuss conservation of linear momentum using Lagrangian formalism.

Q-5

(a) Discuss principle of least action. (06)

(b) Lagrangian for a system is given by $L = \frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2) - \frac{k}{r}$. Obtain Hamilton's equations of motion. (06)

OR

(b) Discuss Routhian procedure giving a suitable example.

Q-6

(a) Define a canonical transformation. Derive symplectic condition for canonical transformation. (06)

(b) Let u(q, p, t) and v(q, p, t) be constants of motion. Show that [u, v] is also a constant of motion. (06)

OR

(b) Show that Poisson brackets are invariant under a canonical transformation.

****