SEAT NO._____ No. of printed pages 2

[76]

Sardar Patel University

M.Sc.(Sem-I), PS01EMTH01, Graph Theory I;
Monday, 1st April, 2019;10.00 a.m. to 01.00 p.m.

Maximum Marks : 70

Note: (i) Notations and terminologies are standard; (ii) Figures to the right indicate marks.

Q.1 Choose the most appropriate option in the following questions. [08]

1. For $G = C_6$, if $D = diam(G)$ and $R = rad(G)$, then

   (a) $D = R$ (b) $D = 2R$ (c) $D = 3R$ (d) None of these

2. If G is complete symmetric digraph with n vertices, then $|E(G)| =$

   (a) n (b) $n(n-1)$ (c) n^2 (d) $\frac{n(n-1)}{2}$

3. Let T be a spanning in-tree with root R. Then

   (a) $d^+(R) > 0$ (b) $d^+(R) = 0$ (c) $d^+(R) < 0$ (d) None of these

4. If G is a simple digraph with vertices $\{v_1, v_2, v_3, \ldots, v_n\}$ & e edges, then $\sum_{i=1}^{n} d^+(v_i) =$

   (a) ne (b) e^2 (c) $2e$ (d) e

5. The coefficient c_4 in Chromatic polynomial of K_4 is

   (a) 0 (b) 1 (c) 4 (d) 4!

6. Which of the following graph is not Hamiltonian?

   (a) K_n (b) P_n (c) C_n (d) None of these

7. If $G = P_{2019}$, then

   (a) $\alpha(G) = \beta(G)$ (c) $\alpha'(G) = \beta(G)$
   (b) $\alpha'(G) = \beta'(G)$ (d) None of these

8. If $G = C_6$ and M is a maximum matching in G, then $|M| =$

   (a) 2 (b) 3 (c) 4 (d) None of these

Q.2 Attempt any seven. [14]

1. Define complete graph.
2. Write any four properties of tree.
3. Define spanning out-tree.
4. Define Adjacency matrix of digraph.
5. Find Chromatic number of C_6.
6. What is four color problem?
7. If G is bipartite graph, then show that $\chi(G) = 2$.
8. Write Hall's matching condition.

(P.T.O.)

9. Define perfect matching.

Q.3

(a) Discuss in detail the Teleprinter problem for $r = 3$. [06]

(b) Define the following with an example (1) Eccentricity of vertex, (2) Center of graph and (3) Radius of graph. [06]

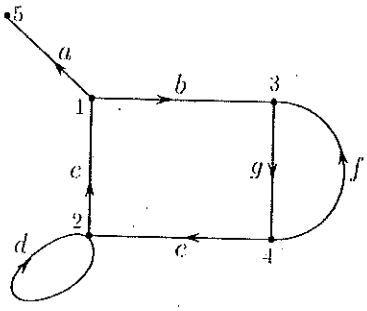
OR

(b) Define the following with an example (1) Symmetric digraph, (2) Asymmetric digraph and (3) Complete Symmetric digraph . [06]

Q.4

(a) Show that the determinant of every square sub matrix of the incidence matrix A of a digraph is $1, -1$ or 0. [06]

(b) Define Adjacency matrix for digraph. Find Adjacency matrix of the following digraph. [06]



OR

(b) Let G be a digraph without self loop. Let A and B denote the incidence matrix and circuit matrix respectively. Suppose that the edges are arranged in the same order in A and B. Show that $AB^T = 0$. [06]

Q.5

(a) Let G be a simple graph with n vertices ($n \geq 3$). If $\delta(G) \geq \frac{n}{2}$, then show that G is Hamiltonian. [06]

(b) Find Chromatic polynomial of $K_{2,2}$. [06]

OR

(b) Suppose G is Hamiltonian graph. Show that for any non-empty $S \subset V(G)$, $c(G-S) \leq |S|$. [06]

Q.6

(a) For $k > 0$, show that every k-regular bipartite graph has a perfect matching. [06]

(b) Define symmetric difference of matchings. Show that every component of the of symmetric difference of two matchings is a path or an even cycle. [06]

OR

(b) Let $G = K_{3,4}$. Then find a
(1) minimal vertex cover of G and $\beta(G)$,
(2) minimal edge cover of G and $\beta'(G)$ and
(3) maximal matching in G and $\alpha'(G)$ [06]