[112]

SEAT No.________

No of printed pages: 2

Sardar Patel University

M.Sc. (Sem-I), PS01CMTH25, Methods of Differential Equations;
Friday, 29^{th} March, 2019; 10.00 a.m. to 01.00 p.m.

Maximum Marks: 70

Note: (i) Notations and terminologies are standard; (ii) Figures to the right indicate marks.

Q.1 Answer the following. [8]

1. The order of differential equation $3y'' + \sqrt{y'} + (\sin x)y = 0$ is
   (A) 1 (B) $\frac{1}{2}$ (C) 2 (D) 3
2. The set of ordinary points of $xy'' + (\cos x)y' + xy = 0$ is
   (A) $\{0\}$ (B) φ (C) $\mathbb{R} \setminus \{0\}$ (D) none of these
3. $\int_{-1}^{1} J_1(x)J_2(x)dx =$
   (A) $\sqrt{\pi}$ (B) -1 (C) 0 (D) none of these
4. $\int_{-1}^{1} x^2 P_3(x)dx =$
   (A) -1 (B) $\frac{1}{5}$ (C) 1 (D) none of these
5. Which of the following is an integrating factor of $dx + dy$?
   (A) $\frac{1}{y^2}$ (B) $\frac{1}{x}$ (C) $\frac{1}{xy}$ (D) none of these
6. Which one is not homogeneous Pfaffian differential equation?
   (A) $x^2dx - yzdy + y^2dz = 0$ (B) $(x+y)dx + (y+z)dy + (z+x)dz = 0$
   (C) $x^2ydx + y^2zdy + z^2dz = 0$ (D) none of these
7. $F(\alpha, \beta; \gamma; 0)$ equals
   (A) 0 (B) $\frac{\alpha\beta}{\gamma}$ (C) 1 (D) none of these
8. The radius of convergence of Gauss's hypergeometric series is
   (A) 0 (B) 2 (C) 1 (D) 3

Q.2 Attempt any ***seven***: [14]

(a) Find the interval of convergence of $\sum_{n=0}^{\infty} \frac{x^n}{n!}$.
(b) Define ordinary point.
(c) State Frobenius theorem.
(d) Define gamma function and find value at 1.
(e) State Fourier-Legendre expansion theorem.
(f) State Picard's theorem.
(g) Find $F(1, \frac{1}{2}; \frac{1}{2}; \frac{1}{2})$.
(h) Find a partial differential equation by eliminating function f from $z = f(\frac{y}{x})$.
(i) State the necessary and sufficient condition that Pfaffian differential equation in three variables is integrable.

(P.T.O.)

(1)

Q.3

(a) Solve: $x^2y'' + \frac{3}{2}xy' - \frac{1}{2}(x+1)y = 0$ near origin. [6]

(b) Solve: $y'' + (x-1)y' + y = 0$ near 1. [6]

OR

(b) Find the general solution of $y'' - xy' - y = 0$ in terms of power series in x.

Q.4

(a) With usual notations, prove that $2\alpha J_\alpha(x) = x[J_{\alpha-1}(x) + J_{\alpha+1}(x)]$. [6]

(b) State Rodrigue's formula and hence find $P_i(x)$, where $i = 0, 1, 2, 3$. [6]

OR

(b) Let $p(x)$ be a polynomial of degree $n \geq 1$ with

$$\int_{-1}^{1} x^k p(x)dx = 0 \quad \text{where} \quad k = 0, 1, 2, ..., n-1,$$

show that $p(x) = cP_n(x)$ for some constant c, where P_n is a Legendre polynomial.

Q.5

(a) Solve $y' - 2(x+xy) = 0,\ y(0) = 1$ using Picard's method of successive approximations. [6]

(b) Find one solution of Gauss's hypergeometric differential equation near origin. [6]

OR

(b) Show that $P_n(x) = F\big(-n, n+1; 1; \frac{1-x}{2}\big)$.

Q.6 [6]

(a) Show that $X \cdot \text{curl}X = 0$ iff $\mu X \cdot \text{curl}(\mu X) = 0$ where $X = (P, Q, R)$ and $P, Q, R, \mu(\neq 0)$ are functions of x, y and z.

(b) Solve: $(x^2 + y^2)p + 2xyq = (x+y)z$. [6]

OR

(b) Verify that the differential equation $x(y^2-1)dx + y(x^2-z^2)dy - z(y^2-1)dz = 0$ is integrable and find its primitive.

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(2)