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[28] Seat No. ________

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SARDAR PATEL UNIVERSITY

M.Sc. (Mathematics) Semester - I Examination

Friday, 22nd March, 2019

PS01CMTH22, Topology - I

Time: 10:00 a.m. to 01:00 p.m. **Maximum marks: 70**

Note: (1) Figures to the right indicate marks of the respective question.
(2) Assume usual/standard notations wherever applicable.

Q-1 Choose the most appropriate option for each of the following questions. [08]

1. A function on ______ space is continuous.
   (a) an indiscrete (b) a discrete (c) a cofinite (d) a metric
2. Boundary of $\mathbb{Q}$ is ______ in usual topology of $\mathbb{R}$.
   (a) $\mathbb{Q}$ (b) $\mathbb{R} \setminus \mathbb{Q}$ (c) $\mathbb{R}$ (d) $\emptyset$
3. Homeomorphic image of a T_2 space is ______.
   (a) T_1 (b) compact (c) closed (d) not T_2
4. Projections are ______ map.
   (a) open (b) constant (c) closed (d) bounded
5. Cardinality of a non-empty connected subset of $\mathbb{Q}$ is ______.
   (a) 0 (b) exactly 1 (c) > 1 (d) infinite
6. The open interval $(0, 1)$ is compact in ______ topology.
   (a) cocountable (b) usual (c) cofinite (d) discrete
7. Every metric space need not be ______.
   (a) T_2 (b) T_4 (c) normal (d) separable
8. A compact ______ space is normal.
   (a) T_2 (b) T_1 (c) bounded (d) none of these

Q-2 Attempt *any seven* of the following. [14]

(a) Prove that $\{(a, b) \mid a, b \in \mathbb{R}\}$ is a base for some topology on $\mathbb{R}$.

(b) Let X be a topological space and $A \subset X$. Prove that if A is closed, then $A = \overline{A}$.

(c) Prove or disprove: $A^\circ \cup B^\circ = (A \cup B)^\circ$.

(d) Show that projections are continuous.

(e) Show that $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2x + 1$ is a homeomorphism.

(f) Prove that a singleton set is connected.

(g) Define a separable topological space and give an example of it.

(h) Define a normal topological space.

(i) State Urysohn's lemma.

(P.T.O)

(1)

Q-3 (a) Let X be a topological space. Prove that arbitrary intersection and finite union of closed sets is closed. [06]

(b) Let X be a Hausdorff space. Prove that a sequence $\{x_n\}$ in X can converge to at most one point in X. [06]

OR

(b) Prove that a topological space X is T_1 if and only if every singleton subset of X is closed in X. [06]

Q-4 (a) Let X and Y be two topological spaces, $f : X \to Y$ be a function and $A, B \subset X$ be two open sets such that $A \cup B = X$. If $f_{|A}, f_{|B}$ are continuous, then prove that f is continuous. [06]

(b) Prove that homeomorphic image of a T_1 space is T_1. [06]

OR

(b) Let X, Y be topological spaces and $f : X \to Y$ be a continuous function. Prove that for each $x \in X$ and for each neighbourhood V of $f(x)$, there is a neighbourhood U of x such that $f(U) \subset V$. [06]

Q-5 (a) Prove that (a, b), $a, b \in \mathbb{R}$ is connected. [06]

(b) Let X be a compact topological space. If Y is a closed subspace of X, then prove that Y is compact. [06]

OR

(b) Let X be a topological space. Prove that X is compact if and only if every family of closed subsets of X with finite intersection property has non-empty intersection. [06]

Q-6 (a) State and prove Cantor's intersection theorem. [06]

(b) Prove that a topological space X is T_4 if and only if for every open set $G \subset X$ and a closed set $E \subset G$, there exists an open set $H \subset X$ such that $E \subset H \subset \overline{H} \subset G$. [06]

OR

(b) State and prove Baire's category theorem for complete metric spaces. [06]

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