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SEAT NO.______ No. of printed pages 2

(27)

Sardar Patel University
Mathematics
PS01CMTH21 Complex Analysis-I

Time: 10.00 a.m. to 01.00 p.m. Total Marks: 70

M.Sc. Ist Semester Date: 19-03-2019

Tuesday

Q.1 Choose the most appropriate option in the following questions. [08]

1. If $z = -\dfrac{1}{2019^{2019}}$, then $Arg(z) =$ _______.

   (a) 0 (b) π (c) 2π (d) $\frac{\pi}{2}$

2. $\lim_{z\to\infty} f(z) = \infty$ if and only if $\lim_{z\to 0} \dfrac{1}{f(\frac{1}{z})} =$ _______.

   (a) 1 (b) 0 (c) ∞ (d) None of these

3. The set of singularities of the function $f(z) = \dfrac{z^3+4}{(z^2-3)(z^2+1)}$ is

   (a) $\{\pm 3, \pm 1\}$ (b) $\{3, 1\}$ (c) $\{\pm\sqrt{3}, \pm i\}$ (d) None of these

4. Which of the following is not a harmonic function ?

   (a) $u(x,y) = \dfrac{y}{x^2+y^2}$

   (b) $u(x,y) = x^2 - y^2$

   (c) $u(x,y) = e^{2019x}$

   (d) None of these

5. If C is the unit circle taken in the positive direction, then $\int_C \frac{1}{\bar{z}} dz =$ _______.

   (a) $2\pi i$ (b) 0 (c) 1 (d) None of these

6. Let $f(z) = \dfrac{7z^6 + 5z^4 + 3z^2 + 1}{z^3}$. If C is the ellipse whose equation in $\mathbb{R}^2$ is $\frac{x^2}{4} + \frac{y^2}{9} = 1$ oriented counterclockwise, then $\int_C f(z)dz =$ _______.

   (a) πi (b) $2\pi i$ (c) 0 (d) None of these

7. $\int_{|z|=3} \frac{\exp(-z)}{z^2} dz =$ _______.

   (a) $2\pi i$ (b) $-2\pi i$ (c) 0 (d) None of these

8. Let T be a linear fraction transformation such that $T(\infty) = 0$, $T(i) = i$, and $T(0) = \infty$. Then

   (a) T is a constant map

   (b) T must be identity map

   (c) no such T exists

   (d) None of these

Q.2 Attempt any seven. [14]

1. If $z \in \mathbb{C}$, then show that $|\text{Re } z| + |\text{Im } z| \le \sqrt{2}|z|$.
2. If a function f is continuous and nonzero at a point z_0, then show that $f(z) \neq 0$ throughout some neighborhood of that point.
3. When is $z_0 \in \mathbb{C}$ called a singularity of f? Determine the singularities of $\frac{1}{z}$.
4. State Integral Inequality.

(P.T.O)

5. Is the Mean Value Theorem true for complex valued functions of real variable? Justify your answer.

6. State Green's Theorem.

7. Define Mobius transformation.

8. Define Cross ratio.

9. If the origin is a fixed point of a linear fractional transformation, then prove that the transformation can be written in the form $w = \frac{z}{Cz+D}$, where $D \neq 0$.

Q.3

(a) Let $f = u + iv$ be defined in a neighbourhood of $z_0 = x_0 + iy_0$. If the partial derivatives u_x, u_y, v_x, v_y exist everywhere in the neighbourhood, they are continuous at (x_0, y_0), $u_x(x_0, y_0) = v_y(x_0, y_0)$ and $u_y(x_0, y_0) = -v_x(x_0, y_0)$, then show that f is differentiable at z_0. [06]

(b) Let f be differentiable at z_0 and g be differentiable at $f(z_0)$. Show that$g o f$ is differentiable at z_0. [06]

OR

(b) Suppose that $\lim_{z \to z_0} f(z) = w_0 \in \mathbb{C}$. Show that (1) f is a bounded in a neighborhood of z_0. (2) For given $\epsilon > 0$ there is $\delta > 0$ such that $|f(z_1) - f(z_2)| < \epsilon$ whenever $0 < |z_1 - z_0| < \delta$ and $0 < |z_2 - z_0| < \delta$. [06]

Q.4

(a) Suppose that f is analytic in a domain D. Prove the following statements (i) If $|f|$ is constant on D, then f is a constant map. (ii) If f is real valued, then f is constant map. [06]

(b) Define harmonic conjugate of a harmonic function u. Construct an analytic function having the imaginary part $v(x, y) = e^{2x} \sin 2y - y$. [06]

OR

(b) Define harmonic conjugate of a harmonic function u. Construct an analytic function having the real part $u(x, y) = y^3 - 3x^2y$. [06]

Q.5

(a) Suppose that $|f(z)| \leq |f(z_0)|$ at each point z in some neighborhood $|z - z_0| < \epsilon$ in which f is analytic. Then show that f has the constant value $f(z_0)$ throughout that neighborhood. [06]

(b) Let f be analytic within and on a simple closed contour C. Show that f is differentiable on the interior of C and $f'(z_0) = \frac{1}{2\pi i} \int_C \frac{f(z)}{(z - z_0)^2} dz$, for all z_0 in the interior to C. [06]

OR

(b) Let f be an entire function. If the real part of f is bounded above, then show that f is a constant map. State the results you use. [06]

Q.6

(a) State and prove Taylor's theorem. [06]

(b) Evaluate $\int_0^\infty \frac{2x^2 - 1}{x^4 + 5x^2 + 4} dx$. [06]

OR

(b) Evaluate $\int_0^\infty \frac{\cos 3x}{(x^2 + 1)^2} dx$. [06]

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