111 SEAT No._______ No of printed pages: 2

Sardar Patel University

M.Sc. (Sem-I), PS01CMTH05, Methods of Differential Equations;
Friday, 29^{th} March, 2019; 10.00 a.m. to 01.00 p.m.

Maximum Marks: 70

Note: (i) Notations and terminologies are standard; (ii) Figures to the right indicate marks.

Q.1 Answer the following. [8]

1. The degree of a differential equation $y'' - (y')^3 = y^3$ is
   (A) 0 (B) 3 (C) 2 (D) 1
2. The set of singular points of $xy'' - (\cos x - 1)y' + xy = 0$ is
   (A) $\{0\}$ (B) φ (C) $\{1\}$ (D) none of these
3. $\Gamma(-\frac{1}{2}) =$
   (A) $2\sqrt{\pi}$ (B) $-2\sqrt{\pi}$ (C) $\sqrt{\pi}$ (D) none of these
4. $\int_{-1}^{1} xP_2(x)dx =$
   (A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{15}$ (D) none of these
5. Which of the following is an integrating factor of $xdx - ydy$?
   (A) $\frac{1}{y}$ (B) $\frac{1}{x}$ (C) $\frac{1}{y^2}$ (D) none of these
6. The differential equation obtained from $az + b = a^2x + y$ by eliminating a and b, is
   (A) $p^2q = 0$ (B) $p = q$ (C) $pq = 1$ (D) none of these
7. $F(\frac{1}{2}, \frac{1}{2}; \frac{1}{2}; \frac{1}{2}) =$
   (A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) 2 (D) none of these
8. The radius of convergence of Gauss's hypergeometric series is
   (A) 0 (B) 1 (C) 2 (D) $\frac{1}{2}$

Q.2 Attempt any ***seven***: [14]

(a) Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{n!}{n^n} x^{2n}$.
(b) Define analytic function.
(c) Show that between any two positive roots of J_3 there is a root of J_2.
(d) Show that $\Gamma(x + 1) = x\Gamma(x)$, $x > 0$.
(e) Show that $P_n(-x) = (-1)^n P_n(x)$, $n \in \mathbb{N} \cup \{0\}$.
(f) State Picard's theorem.
(g) Find a partial differential equation by eliminating α and β from $z = \alpha x + \beta y$.
(h) Find $F(1, 1; 2; x)$.
(i) State Gauss's formula.

(P.T.O.)

(1)

Q.3

(a) Solve: $y'' - xy = 0$ near 0. [6]

(b) Classify singularities: $x(x-1)^2(x+2)y'' + x^2(x-1)y' + (x+2)y = 0$ [6]

OR

(b) Find series solution of $3xy'' + 2y' + y = 0$ near 0.

Q.4

(a) State and prove Rodrigue's formula. [6]

(b) Prove: $\frac{d}{dx}[x^{-\alpha}J_\alpha(x)] = -x^{-\alpha}J_{\alpha+1}(x)$. [6]

OR

(b) Show that $x^2 = \sum_{n=1}^{\infty} \frac{2}{\lambda_n J_3(\lambda_n)} J_2(\lambda_n x)$, $x \in (0,1)$, where $\{\lambda_n\}$ is a sequence of positive roots of $J_2(x)$.

Q.5

(a) Solve $y' = y$, $y(0) = 1$ using Picard's method of successive approximations. [6]

(b) Find a necessary and sufficient condition that there exists between two functions $u(x,y)$ and $v(x,y)$, a relation $F(u,v) = 0$ not involving x or y explicitly. [6]

OR

(b) Verify that the differential equation $(y^2 + yz)dx + (z^2 + zx)dy + (y^2 - xy)dz = 0$ is integrable and find its primitive.

Q.6

(a) Show that $P_n(x) = F\big(-n, n+1; 1; \frac{1-x}{2}\big)$. [6]

(b) Solve: $z^2 = pqxy$ using Charpit's method. [6]

OR

(b) Explain Jacobi's method.

X

(2)