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Seat No. ________

[27]

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SARDAR PATEL UNIVERSITY

M.Sc. (Mathematics) Semester - I Examination

Friday, 22nd March, 2019

PS01CMTH02, Topology - I

Time: 10:00 a.m. to 01:00 p.m. **Maximum marks: 70**

Note: (1) Figures to the right indicate marks of the respective question.
(2) Assume usual/standard notations wherever applicable.

Q-1 Choose the most appropriate option for each of the following questions. [08]

1. A function whose codomain is ________ space is continuous.
   (a) an indiscrete (b) a discrete (c) a cofinite (d) a metric
2. Interior of $\mathbb{Q}$ is ________ in usual topology of $\mathbb{R}$.
   (a) $\mathbb{Q}$ (b) $\mathbb{R}\setminus\mathbb{Q}$ (c) $\mathbb{R}$ (d) $\emptyset$
3. Finite product of T_2 spaces ________.
   (a) is T_1 (b) is compact (c) is closed (d) need not be T_2
4. A complete metric space is compact if it is ________.
   (a) totally bounded (c) Hausdorff
   (b) bounded (d) connected
5. $[a, b]$, where $a, b \in \mathbb{R}$, $a < b$ is not compact in ________ topology.
   (a) indiscrete (b) discrete (c) usual (d) cofinite
6. Closed subset of a ________ space is compact.
   (a) T_2 (b) T_1 (c) compact (d) connected
7. A metric space need not be ________.
   (a) T_2 (b) T_4 (c) regular (d) separable
8. A ________ subset of a Hausdorff space is normal.
   (a) compact (b) connected (c) bounded (d) closed

Q-2 Attempt *any seven* of the following. [14]

(a) Prove that $\{(-n, n) \mid n \in \mathbb{N}\}$ is a base for some topology on $\mathbb{R}$.

(b) Let X be a topological space and $A \subset X$. Prove that if A is open, then $A = A^\circ$.

(c) Prove or disprove: $\overline{(A \cap B)} = \overline{A} \cap \overline{B}$.

(d) Define totally bounded metric space.

(e) Show that $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x - 2$ is a homeomorphism.

(f) State Heine-Borel Theorem.

(g) When is a subset A of a topological space $(X, \mathscr{T})$ called compact?

(h) Define a separable topological space and give an example of it.

(i) Define a normal topological space.

(1) (P.T.O.)

Q-3 (a) State and prove Pasting lemma. [06]

(b) Let $\mathscr{T}, \mathscr{T}'$ be two topologies on a set X generated by the bases $\mathscr{B}, \mathscr{B}'$ respectively. Prove that $\mathscr{T}'$ is finer than $\mathscr{T}$ if and only if for every $B \in \mathscr{B}$ and for every $x \in B$, there exists $B' \in \mathscr{B}'$ such that $x \in B' \subset B$. [06]

OR

(b) Prove that a topological space X is T_1 if and only if every singleton subset of X is closed in X. [06]

Q-4 (a) State and prove Cantor's intersection theorem. [06]

(b) Prove that projections are open and continuous maps. [06]

OR

(b) Let $(X_i, \mathscr{T}_i)$ be topological spaces and $X = \prod_{i=1}^{n} X_i$ be the product space with product topology. Prove that X is T_1 if and only if each X_i is T_1. [06]

Q-5 (a) Show that $[0, 1]$ is compact. [06]

(b) Prove that compact subset of a Hausdorff space is closed. [06]

OR

(b) Let X be a topological space. Prove that X is compact if and only if every family of closed subsets of X with finite intersection property has non-empty intersection. [06]

Q-6 (a) Prove that a topological space X is T_4 if and only if for every open set $G \subset X$ and a closed set $E \subset G$, there exists an open set $H \subset X$ such that $E \subset H \subset \overline{H} \subset G$. [06]

(b) Prove that every second countable topological space is separable. [06]

OR

(b) Prove that every metric space is normal. [06]

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(2)