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SC

SEAT NO._______

No. of printed pages 2

Sardar Patel University
Mathematics
PS01CMTH01 Complex Analysis-I

Time: 10.00 a.m. to 01.00 p.m. **Total Marks: 70**

M.Sc. Ist Semester **Date: 19-03-2019**

Tuesday

Q.1 Choose the most appropriate option in the following questions. [08]

1. The equation $|z-4i|+|z+4i|=10$ represents a

   (a) circle (b) ellipse (c) hyperbola (d) None of these

2. If C is any n^{th} root of unity other than unity, then $1+C+C^2+\cdots+C^{n-1}=$ _______.

   (a) 2^n (b) n (c) 0 (d) None of these

3. Let $f(z)=\bar{z}, z\in\mathbb{C}$. Then

   (a) f is differentiable at 0
   (b) f is differentiable on $\mathbb{C}$
   (c) f is differentiable on $\mathbb{C}\setminus\{0\}$
   (d) f is nowhere differentiable

4. Which of the following is not a harmonic function ?

   (a) $u(x,y)=\dfrac{y}{x^2+y^2}$
   (b) $u(x,y)=x^2-y^2$
   (c) $u(x,y)=e^{2019x}$
   (d) None of these

5. Let C be the positively oriented circle $|z|=2$. Then $\int_C \dfrac{z}{(9-z^2)(z+i)}dz=$ _______.

   (a) $2\pi i$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{5}$ (d) None of these

6. If C is the unit circle taken in the positive direction, then $\int_C \dfrac{1}{z}dz=$ _______.

   (a) $2\pi i$ (b) 0 (c) 1 (d) None of these

7. The set of singularities of the function $f(z)=\dfrac{z^3+4}{(z^2-3)(z^2+1)}$ is

   (a) $\{\pm 3, \pm 1\}$ (b) $\{3, 1\}$ (c) $\{\pm\sqrt{3}, \pm i\}$ (d) None of these

8. Let T be a linear fraction transformation such that $T(\infty)=0$, $T(i)=i$, and $T(0)=\infty$. Then

   (a) T is a constant map
   (b) T must be identity map
   (c) no such T exists
   (d) None of these

Q.2 Attempt any seven. [14]

1. Find the locus of $|z-1|=|z+i|$.
2. Find the Principal Argument of $z=\frac{-2}{1+\sqrt{3}i}$.
3. When is $z_0\in\mathbb{C}$ called a singularity of f? Determine the singularities of $\frac{1}{z}$.
4. Write Cauchy-Riemann equations in polar coordinates.
5. State Liouville's Theorem.

(P.T.O.)

6. State Maximum modulus principle.
7. Define simple closed contour with example.
8. State Cauchy Residue Theorem.
9. Find the Laurent's series of $f(z) = \frac{1}{(z-1)(z-2)}$ in the region $|z| > 2$.

Q.3

(a) Suppose $\theta_1, \theta_2, \theta_3 \ldots.., \theta_n$ are real numbers. Show that [06]

$$\prod_{j=1}^{n}(\cos\theta_j + i\sin\theta_j) = \cos(\sum_{j=1}^{n}\theta_j) + i\sin(\sum_{j=1}^{n}\theta_j)$$

(b) If z_1 and z_2 are complex numbers, then show that [06]
(1) $|z_1 + z_2| \leq |z_1| + |z_2|$. (2) $||z_1| - |z_2|| \leq |z_1 - z_2|$.

OR

(b) Let z_1 and z_2 be nonzero complex numbers. Show that arg $(z_1 z_2) =$ arg $(z_1)+$ arg (z_2). [06]

Q.4 [06]

(a) Obtain the necessary condition for the existence of derivative of a function at a point. [06]

(b) Define harmonic conjugate of a harmonic function u. Construct an analytic function having the imaginary part $v(x, y) = e^{2x}\sin 2y - y$. [06]

OR

(b) Define harmonic conjugate of a harmonic function u. Construct an analytic function having the real part $u(x, y) = y^3 - 3x^2y$. [06]

Q.5

(a) State and prove fundamental theorem of algebra. [06]

(b) Let f be analytic within and on a simple closed contour C, taken in positive sense. If z_0 is any interior to C, then show that $f(z) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z-z_0}dz$ [06]

OR

(b) Let f be analytic within and on a simple close contour C. Show that f is differentiable on the interior of C and $f'(z_0) = \frac{1}{2\pi i}\int_C \frac{f(z)}{(z-z_0)^2}dz$, for all z_0 in the interior to C. [06]

Q.6

(a) State and prove Taylor's theorem. [06]

(b) Evaluate $\int_0^\infty \frac{\sin x}{x}dx$. [06]

OR

(b) Evaluate $\int_0^\infty \frac{2x^2 - 1}{x^4 + 5x^2 + 4}dx$. [06]

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