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SARDAR PATEL UNIVERSITY

M.Sc. (Semester - I) Examination
Tuesday April 24, 2018
Time: 10:00 a.m. to 01:00 p.m.
Subject: Mathematics
Course No. PS01EMTH22 (Mathematical Classical Mechanics)

Note: (1) All the questions are to be answered in the answer book only. Total Marks: 70
(2) Figures to the right indicate marks of the respective question.
(3) Assume standard notations wherever applicable.

Q-1 Choose the most appropriate option for each of the following questions: (08)

1. A particle moving on the surface of a sphere is ______ constraint.
   (a) a rheonomic (c) a holonomic
   (b) a non-holonomic (d) not a
2. Lagrangian of a system of particles is ______.
   (a) not unique (b) unique (c) constant (d) zero
3. The condition for extremum of the integral $J = \int_{x_1}^{x_2} f(\dot{y}, x)\, dx$ is ______.
   (a) $\frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}}\right) = 0$ (c) $\frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}}\right) - \frac{\partial f}{\partial \dot{y}} = 0$
   (b) $\frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}}\right) - \frac{\partial f}{\partial x} = 0$ (d) $\frac{d}{dy}\left(\frac{\partial f}{\partial \dot{x}}\right) - \frac{\partial f}{\partial x} = 0$
4. If Lagrangian L does not depend on q_j explicitly, then ______ is conserved.
   (a) L (b) H (c) q_j (d) p_j
5. Which of the following is correct?
   (a) $\frac{\partial L}{\partial t} = \frac{\partial H}{\partial t}$ (b) $q_j = \frac{\partial H}{\partial \dot{p}_j}$ (c) $\dot{q}_j = \frac{\partial H}{\partial p_j}$ (d) $p_j = -\frac{\partial H}{\partial \dot{q}_j}$
6. In matrix form of Hamilton's equations of motion, $\dot{\eta} =$ ______.
   (a) $J\frac{\partial H}{\partial \eta}$ (b) $J'\frac{\partial H}{\partial \eta}$ (c) $\frac{\partial H}{\partial \eta}J$ (d) $-J\frac{\partial H}{\partial \eta}$
7. For symplectic matrices M and N, the matrix ______ need not be symplectic.
   (a) MN (b) $M+N$ (c) $M^{-1}N$ (d) MN^{-1}
8. $[p_2, q_1] =$ ______; notations being usual.
   (a) 0 (b) 1 (c) -1 (d) none of these

Q-2 Answer *any seven* of the following: (14)

(a) Define a non-holonomic constraint and give its example.
(b) What is a rigid body? State degrees of freedom in rigid body with more than three particles.
(c) Define action integral.
(d) State the condition for extremum of the integral $\int_{t_1}^{t_2} L(q, \dot{q}, t)\, dt$?
(e) Define a cyclic coordinate.
(f) In usual notations show that $\frac{\partial H}{\partial t} = \frac{dH}{dt}$.
(g) State Routhian equations of motion for a system with n-degrees of freedom.

(P.T.O.)

(h) Show that Poisson bracket is linear in the first variable.

(i) State the form of generating functions of type F_1 and F_2.

Q-3 (a) State Lagrange's equations of motion in general form and hence derive the form when the forces are conservative and potential is independent of velocities. (06)

(b) Derive Lagrange's equations of motion for a spherical pendulum. (06)

OR

(b) Let L be Lagrangian of a system with n-degrees of freedom and $L' = L + \frac{dF}{dt}$, where $F(q_1, q_2, \ldots, q_n, t)$ is an arbitrary differentiable function of its arguments. Show that L' satisfies Lagrange's equations of motion. (06)

Q-4 (a) State and prove the law of conservation of total energy in Lagrangian formalism. (06)

(b) Let $L = \frac{I_1}{2}(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta) + \frac{I_3}{2}(\dot{\psi} + \dot{\phi}\cos\theta)^2 - mgl\,cos\theta$ be the Lagrangian of a system. Compute the energy function. Is it conserved? Why? (06)

OR

(b) Lagrangian of a system is given by $L = \frac{m}{2}(\dot{x}^2 + \dot{y}^2) - (x^2 + y^2)^{\frac{1}{2}}$. How many generalized coordinates are there? Which of them are cyclic? Compute the generalized momenta. Is any of them conserved? Justify. (06)

Q-5 (a) State Hamilton's modified principle and hence derive Hamilton's equations of motion from it. (06)

(b) Describe Routhian procedure and obtain Routhian of a system with Lagrangian $L = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{k}{r}$. (06)

OR

(b) Let $L = \frac{1}{2}m\left(\dot{q}^2 - k^2q^2\right)$ be Lagrangian of a system, where m and k are constants. Obtain the corresponding Hamiltonian and hence derive Hamilton's equations of motion. (06)

Q-6 (a) Derive the symplectic condition for a transformation to be canonical (06)

(b) Define canonical transformation. Check whether $Q = \log\left(\frac{\sin p}{q}\right)$, $P = q\cot p$ is a canonical transformation or not. (06)

OR

(b) Describe the method of obtaining formal solution of a mechanical problem using Poisson bracket formalism. (06)

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