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SEAT No.________

[13]

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SARDAR PATEL UNIVERSITY

M. Sc. (Semester I) Examination

Date: 24-04-2018 **Time: 10.00 To 01.00**

Subject: MATHEMATICS **Paper No. PS01EMTH21 – (Graph Theory – I)**

Total Marks: 70

1. Choose the correct option for each question: [8]

(1) If $K_{1,n} = K_{n+1}$, then

(a) n = 1 (b) n = 2 (c) n > 2 (d) none of these

(2) A symmetric digraph is

(a) Euler (b) connected (c) regular (d) balanced

(3) For $G = C_5$ with anticlockwise direction, rank(B) is

(a) 5 (b) 4 (c) 1 (d) none of these

(4) Let T be a spanning in-tree with root R. Then

(a) $d^-(R) = 0$ (b) $d^-(R) > 0$ (c) $d^+(R) > 0$ (d) none of these

(5) The coefficient c_2 in chromatic polynomial of C_7 is

(a) 0 (b) 1 (c) 7 (d) 7!

(6) If G is a planar graph, then $\chi(G)$

(a) = 4 (b) ≤ 4 (c) 5 (d) none of these

(7) Let G be a simple graph without isolated vertex. Then a matching M in G is

(a) maximum $\Rightarrow$ perfect (c) maximum $\Rightarrow$ maximal
(b) maximal $\Rightarrow$ maximum (d) maximum $\Rightarrow$ perfect

(8) For which of the following graphs, $\alpha'(G) = \beta(G)$?

(a) C_9 (b) K_{10} (c) K_{11} (d) P_{12}

2. Attempt any SEVEN: [14]

(a) Find the diameter of $K_{m,n}$ $(m, n \geq 2)$.
(b) Prove or disprove: An Euler digraph is connected.
(c) Define adjacency matrix in a digraph and give one example of it.
(d) Prove or disprove: Every connected digraph has a spanning out-tree.
(e) Prove or disprove: If G is a bipartite graph, then it is a tree.
(f) Define uniquely colourable graph.
(g) Why P_4 is not isomorphic to $K_{1,3}$?
(h) Define an edge cover of a graph and give one example of it.
(i) Prove or disprove: The graph P_8 has a perfect matching.

(P.T.O.)

3. (a) Define the following with examples: [6]
(i) Asymmetric digraph (ii) Symmetric digraph (iii) Strongly connected digraph

(b) Prove: An arborescence is a tree in which every vertex other than the root has an in-degree exactly one. [6]

OR

(b) Obtain De Bruijn cycle for r = 3 with all detail. [6]

4. (a) Let G be a connected digraph with n vertices. Prove that rank of $A(G) = n - 1$. [6]

(b) Prove that for each $n \geq 1$, there is a simple digraph with n vertices $v_1, v_2, \ldots, v_n$ such that $d^+(v_i) = i - 1$ and $d^-(v_i) = n - i$ for each $i = 1, 2, \ldots n$. [6]

OR

(b) Define spanning in-tree, spanning out-tree & give one example of each in a single digraph. [6]

5. (a) Prove: If G is Hamiltonian, then, for each $S \subset V(G)$, $c(G - S) \leq |S|$. [6]

(b) Prove: If a graph does not contain an odd cycle, then it is 2-chromatic. [6]

OR

(b) Find the coefficients c_2 and c_3 of Chromatic polynomial of graph C_4. [6]

6. (a) Let G be a graph (no isolated vertex) with n vertices. Prove that $\alpha'(G) + \beta'(G) = n$. [6]

(b) Prove: Every component of a symmetric difference of two matching is either a path or a cycle of even length. [6]

OR

(b) Define $\alpha'(G)$, $\beta(G)$ and find it with the corresponding sets for $G = K_5$. [6]

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