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SEAT No.________

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SARDAR PATEL UNIVERSITY

M.Sc. (Semester-I) Examination

April – 2018

Tuesday 24/04/2018

Time: 10:00 AM to 1:00 PM

Subject: Mathematics **Course No.PS01EMTH02**

Mathematical Classical Mechanics

Note:
(1) All questions (including multiple choice questions) are to be answered in the answer book only.
(2) Numbers to the right indicate full marks of the respective question.

Total Marks: 70

Q-1 Choose most appropriate answer from the options given. (08)

(1) For a system of N particles having n degrees of freedom
(a) $n \leq N$ (b) $N \leq n$ (c) $n = N$ (d) none of these is true

(2) The motion of a particle on a sphere is______ constraint.
(a) not a (b) a holonomic (c) non-holonomic (d) conservative

(3) The condition for extremum of $I = \int_{x_1}^{x_2} f(y, \dot{y}, x)dx$ is ______
(a) $\frac{d}{dx}\left(\frac{\partial f}{\partial y}\right) = 0$ (b) $\frac{\partial f}{\partial x} = constant$
(c) $\frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}}\right) - \frac{\partial f}{\partial y} = 0$ (d) $\left(\frac{\partial f}{\partial \dot{y}}\right) = constant$

(4) If $\frac{\partial L}{\partial t} = 0$ then __________ is conserved
(a) p_j (b) h (c) $\dot{p}_j$ (d) E

(5) Which one of the following is correct?
(a) $\frac{\partial H}{\partial p_j} = 0$ (b) $H = E$ (c) $\frac{dH}{dt} = -\frac{\partial L}{\partial t}$ (d) none of these

(6) If Lagrangian does not depend on a coordinate q_j explicitly then
(a) $\frac{\partial H}{\partial q_j} = 0$ (b) $p_j = 0$ (c) $\frac{\partial H}{\partial p_j} = 0$ (d) $\frac{\partial H}{\partial q_j} \neq 0$

(7) For the Jacobian matrix M for a canonical transformation pick the correct statement
(a) M is Identity (b) $|M| = 0$ (c) M is symplectic (d) M is singular

(8) In usual notations, $[p_1, q_1] =$ ________.
(a) 0 (b) -1 (c) 1 (d) zero matrix

Q-2 Answer any **Seven.** (14)

(1) Give an example of holonomic constraint.
(2) State Lagrange's equations of motion in the case of velocity dependant potential.
(3) State Euler-Lagrange equations and hence derive condition for extremum of $J = \int_{x_1}^{x_2} f(\dot{y}_1, \dot{y}_2, \dots \dot{y}_n, x)dx$.
(4) Define generalized momentum conjugate to a generalized coordinate. When it is conserved?

(5) State Hamilton's equations of motion in matrix form.
(6) Is it true that inverse of a canonical transformation is also a canonical transformation?
(7) State transformation equations for a generating function of type F_1.
(8) Define Lagrange bracket.
(9) Find $[u, v]$ for $u = q_1 q_2,\ v = p_1 p_2$.

Q-3

(a) State Lagrange's equations in general form and hence derive the form in the case of frictional forces. (06)

(b) Giving all details obtain Lagrange's equations of motion for Atwood's machine. (06)

OR

(b) Obtain Lagrangian for a spherical pendulum.

Q-4

(a) Discuss conservation of total energy using Lagrangian formalism. (06)

(b) Obtain the curve for minimum surface of revolution using variational principle. (06)

OR

(b) Lagrangian of a system is given by $L = \frac{1}{2}(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{1}{r^2}$. Obtain expressions of all generalized momenta, energy function. Which of them are conserved?

Q-5

(a) State Hamilton's modified principle. Using it derive Hamilton's equations of motion. (06)

(b) State and prove principle of least action. (06)

OR

(b) Obtain Hamiltonian corresponding to the Lagrangian

$$L = a\,\dot{x}^2 + b\frac{\dot{y}}{x} + c\dot{x}\dot{y} + fy^2\dot{x}\dot{z} + g\dot{y} - k\sqrt{x^2 + y^2}$$

Q-6

(a) Define Poisson bracket. State the matrix form of Poisson bracket and show that they are invariant under a canonical transformation. (06)

(b) Show that the transformation, (06)

$Q = \log\left(1 + \sqrt{q}\cos p\right), P = 2\sqrt{q}\left(1 + \sqrt{q}\cos p\right)\sin p$

is canonical.

OR

(b) Hamiltonian for a motion in one dimension with constant acceleration a is given by $H = \frac{p^2}{2m} - max$. Using Poisson bracket formalism obtain the expression for x subject to the conditions $x = x_0, p = p_0$ at $t = 0$.

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