SEAT No.________

[21]

No of printed pages: 2

Sardar Patel University

M.Sc. (Sem-I), PS01CMTH25; Methods of Differential Equations; (NC)
Saturday, 21^{st} April, 2018; 10.00 a.m. to 01.00 p.m.

Maximum Marks: 70

Note: (i) Notations and terminologies are standard; (ii) Figures to the right indicate marks.

Q.1 Answer the following. [8]

1. The function $f : \mathbb{R} \to \mathbb{R}, f(x) = \dfrac{\cos x - 1}{x}, x \neq 0$ and $f(0) = \frac{1}{3}$ is
   (A) analytic at 0 (B) continuous but not differentiable at 0
   (C) not continuous at 0 (D) none of these
2. The set of singular points of $xy'' + (e^x - 1)y = 0$ is
   (A) $\{0\}$ (B) $\{1\}$ (C) φ (D) none of these
3. $J_3(x) =$
   (A) $-J_{-3}(x)$ (B) $-J_{-3}(-x)$ (C) $J_3(-x)$ (D) none of these
4. $\int_{-1}^{1} xP_1(x)dx =$
   (A) $\frac{2}{3}$ (B) $\frac{1}{3}$ (C) 0 (D) 1
5. Which of the following is an integrating factor of $2xydx + x^2dy$?
   (A) $\frac{1}{x^2}$ (B) $\frac{1}{xy}$ (C) $\frac{1}{y^2}$ (D) none of these
6. Which one is homogeneous Pfaffian differential equation?
   (A) $x^2ydx + y^2xdy + zydz = 0$
   (B) $(x^2+1)dx + (y^2+1)dy + (z^2+1)dz = 0$
   (C) $xydx + yzdy + zdz = 0$
   (D) none of these
7. $F(-1, \frac{1}{2}; \frac{1}{2}; \frac{1}{2}) =$
   (A) 1 (B) 2 (C) -1 (D) none of these
8. $F(\alpha, \beta; \gamma; 1)$ converges if
   (A) $\gamma > \alpha - 2\beta$ (B) $\gamma > \alpha + \beta$
   (C) $\gamma < \alpha + \beta - 1$ (D) $2\alpha < 2\gamma - 1$

Q.2 Attempt any ***seven***: [14]

(a) Find the interval of convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$.
(b) Define ordinary point of a differential equation.
(c) Show that $\Gamma(x+1) = x\Gamma(x)$, where $x > 0$.
(d) Show that between any two positive zeros of J_0 there is a zero of J_1.
(e) State Rodrgue's formula and hence find $P_0(x)$.
(f) State Picard's theorem.
(g) Find $F(\alpha, \beta; \gamma; 0)$.
(h) Find a partial differential equation by eliminating a and b from $z = (x-a)(y-b)$.
(i) Define Pfaffian differential equation in three variables and what is the necessary and sufficient condition that it is integrable ?

[P.T.O.]

Q.3

(a) Solve: $y'' - xy = 0$ near 0. [6]

(b) Classify singularities of $x^4y'' + x^3(x+2)y' + y = 0$ near ∞. [6]

OR

(b) Solve: $2x^2y'' + 3xy' - (x+1)y = 0$ near 0.

Q.4

(a) Prove: $\frac{d}{dx}\left[x^{-\alpha}J_\alpha(x)\right] = -x^{-\alpha}J_{\alpha+1}(x)$. [6]

(b) Prove: $\int_{-1}^{1} P_n(x)P_m(x)dx = 0$ where $m \neq n$. [6]

OR

(b) Find the first four terms of the Fourier-Legendre expansion of the function

$$f(x) = \begin{cases} 0, & -1 \leq x \leq 0 \\ 1, & 0 < x \leq 1. \end{cases}$$

Q.5

(a) Solve $y' - (x+y) = 0,\ y(0) = 1$ using Picard's method of successive approximations. [6]

(b) State and prove integral representation of Gauss's hypergeometric function. [6]

OR

(b) Prove: $P_n(x) = F\left(-n, n+1; 1; \frac{1-x}{2}\right)$.

Q.6

(a) Find a necessary and sufficient condition that there exists between two functions $u(x,y)$ and $v(x,y)$, a relation $F(u,v) = 0$ not involving x or y explicitly. [6]

(b) Solve: $(x^2+y^2)p + 2xyq = (x+y)z$. [6]

OR

(b) Verify that the differential equation $(y^2+yz)dx + (z^2+zx)dy + (y^2-xy)dz = 0$ is integrable and find its primitive.