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Seat No.: ____ No. of printed pages: 2

[31]

Sardar Patel University

M.Sc. (Mathematics) Semester - I Examination (NC)
Monday, 16th April, 2018
PS01CMTH22, Topology-I

Time: 10:00 a.m. to 01:00 p.m. **Maximum marks: 70**

Note: Figures to the right indicate full marks of the respective questions.
Assume standard notations wherever applicable.

Q-1 Write the question number and appropriate **option number only** for each question. [8]

(a) Among the following topologies, ____ topology on $\mathbb{R}$ is the finest T_1-topology.
(i) cofinite (ii) indiscrete (iii) lower limit (iv) usual

(b) A function whose codomain is ____ space is continuous.
(i) indiscrete (ii) cofinite (iii) discrete (iv) metric

(c) A continuous function between metric spaces maps a ____ sequence to ____ sequence.
(i) Cauchy, convergent (iii) bounded, Cauchy
(ii) convergent, Cauchy (iv) Cauchy, bounded

(d) Projections are ____.
(i) continuous (ii) constant (iii) closed (iv) bounded

(e) $[0,1)$ is connected with ____ topology.
(i) discrete (ii) upper limit (iii) lower limit (iv) indiscrete

(f) $\mathbb{R}$ with ____ topology is not T_3.
(i) cocountable (ii) discrete (iii) usual (iv) lower limit

(g) ____ subspace of normal space is T_4.
(i) Any (ii) Compact (iii) Hausdorff (iv) infinite

(h) ____ subspace of a complete metric space is complete.
(i) Any (ii) Compact (iii) Open (iv) infinite

Q-2 Attempt *Any Seven* of the following: [14]

(a) Show that $\{[a,b) : a \in \mathbb{Q}, b \in \mathbb{N}, a < b\}$ is an open base for some topology on $\mathbb{R}$.

(b) Find the interior of $\mathbb{Q}$ in cocountable topology on $\mathbb{R}$.

(c) Consider $\mathbb{R}$ with the cofinite topology. Show that a one-one function $f : \mathbb{R} \to \mathbb{R}$ is continuous.

(d) Show that projections are continuous.

(e) Define *second countable space* and *separable space*.

(f) Let X be a topological space. If X is disconnected, then show that there exists a nonempty, proper $A \subset X$ such that A is clopen in X.

(g) Define a T_3-space and show that a discrete space is T_3.

(h) State Urysohn's Lemma.

(i) If a set X with the discrete topology is second countable, then show that X is countable.

(P.T.O.)

Q-3 (j) Define *open base*. Show that $\mathscr{B}_1 = \{(a,b) : a,b \in \mathbb{Q}, a < b\}$ and $\mathscr{B}_2 = \{(a,b) : a,b \in \mathbb{R}, a < b\}$ generate the same topology on $\mathbb{R}$. [6]

(k) Define *closure of a subset* of a topological space. For a subset A of a topological space X, show that $\overline{A} = A \cup A'$. [6]

OR

(k) Define a T_1-*space* and show that a subspace of a T_1-space T_1. [6]

Q-4 (l) Define *continuous function*. Let X, Y be topological spaces and $f : X \to Y$ be a function. Show that f is continuous if and only if $f^{-1}(E)$ is closed for every closed subset E of Y. [6]

(m) Show that a subset of a metric space is bounded if and only if it is contained in some open sphere. [6]

OR

(m) Define *homeomorphism*. Show that homeomorphic image of a T_1-space is T_1. [6]

Q-5 (n) Show that a topological space X is disconnected if and only if there is a continuous function f from X onto $\{0,1\}$, where $\{0,1\}$ carries the discrete topological space. [6]

(o) Show that every second countable topological space is separable. [6]

OR

(o) Show that a compact metric space is bounded but the converse does not hold. [6]

Q-6 (p) Show that a topological space X is T_4 if and only if for every open set $G \subset X$ and a closed set $E \subset G$, there exists an open set $H \subset X$ such that $E \subset H \subset \overline{H} \subset G$. [6]

(q) Let (X, d) be a metric space and A be a nonempty subset of X. Show that $x \in \overline{A}$ if and only if $d(x, A) = 0$. [6]

OR

(q) State and prove Cantor's Intersection Theorem. [6]

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