SEAT No.________

[20]

No of printed pages: 2

Sardar Patel University

M.Sc. (Sem-I), PS01CMTH05, Methods of Differential Equations;(NC)

Saturday, 21^{st} April, 2018; 10.00 a.m. to 01.00 p.m.

Maximum Marks: 70

Note: (i) Notations and terminologies are standard; (ii) Figures to the right indicate marks.

Q.1 Answer the following. [8]

1. The degree of differential equation $(y'')^2 - x^3(y')^3 = 0$ is
   (A) 1 (B) 2 (C) 3 (D) 0
2. The set of ordinary points of $xy'' + xy' + (e^x - 1)y = 0$ is
   (A) $\{0\}$ (B) φ (C) $\{1\}$ (D) none of these
3. $\Gamma(\frac{1}{2}) =$
   (A) $\sqrt{\pi}$ (B) 0 (C) 1 (D) none of these
4. $\int_{-1}^{1} x^2 P_1(x)dx =$
   (A) 0 (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) none of these
5. Which of the following is not an integrating factor of $ydx + xdy$?
   (A) 2 (B) $\frac{1}{3}$ (C) $\frac{1}{y^2}$ (D) none of these
6. The differential equation obtained from $z = (x+a)(y+b)$ by eliminating a, b, is
   (A) $z = p + q$ (B) $z + pq = 0$ (C) $z = pq$ (D) none of these
7. $F(1, 1; 2; x) =$
   (A) $-\frac{\ln(1-x)}{x}$ (B) $\frac{\ln(1-x)}{x}$ (C) $\ln(1-x)$ (D) none of these
8. $F(-1, \frac{1}{4}; \frac{1}{4}; 0)$ equals
   (A) -1 (B) 0 (C) 4 (D) 1

Q.2 Attempt any ***seven***: [14]

(a) Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{n}{(n+1)} x^n$.
(b) State Frobenius theorem.
(c) Show that $\Gamma(n+1) = n!$ where $n \in \mathbb{N} \cup \{0\}$.
(d) State orthogonality of Bessel's functions.
(e) Prove: $P_n(-x) = P_n(x)$ where n is an even nonnegative integer.
(f) State Picard's theorem.
(g) Find a partial differential equation by eliminating F from $F(x^2 + y^2) + xy = z$.
(h) Find $F(1, \frac{1}{2}; \frac{1}{2}; \frac{1}{2})$.
(i) Find radius of convergence of Gauss's hypergeometric series.

[P.T.O]

Q.3

(a) Solve: $y'' - xy' - y = 0$ near origin. [6]

(b) Solve: $x^2y'' - xy' - (x-1)y = 0$ near 0. [6]

OR

(b) Find the general solution of $y'' + (x-1)y' + y = 0$ in terms of power series in $x - 1$.

Q.4

(a) Prove: $(n+1)P_{n+1} = (2n+1)xP_n - nP_{n-1}$ where $n \in \mathbb{N}$. [6]

(b) Show that between any two consecutive positive zeros of J_0 there is a unique zero of J_1. [6]

OR

(b) Show that $x^2 = \sum_{n=1}^{\infty} \frac{2}{\lambda_n J_3(\lambda_n)} J_2(\lambda_n x)$, $x \in (0,1)$, where $\{\lambda_n\}$ is a sequence of positive roots of $J_2(x)$.

Q.5

(a) Explain Picard's method of successive approximations. [6]

(b) Show that $X \cdot \text{curl} X = 0$ iff $\mu X \cdot \text{curl}(\mu X) = 0$ where $X = (P, Q, R)$ and $P, Q, R, \mu (\neq 0)$ are functions of x, y and z. [6]

OR

(b) Verify that the differential equation $(y^2 + z^2)dx + xydy + xzdz = 0$ is integrable and find its primitive.

Q.6

(a) State and prove Integral representation of Gauss's hypergeometric function. [6]

(b) Solve: $z^2 = pqxy$ using Charpit's method. [6]

OR

(b) Explain Jacobi's method.

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