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Seat No.: ________ No. of printed pages: 2

SARDAR PATEL UNIVERSITY

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M.Sc. (Mathematics) Semester - I Examination (NC)

Monday, 16^{th} April, 2018

PS01CMTH02, Topology-I

Time: 10:00 a.m. to 01:00 p.m. **Maximum marks: 70**

Note: Figures to the right indicate full marks of the respective questions.
Assume standard notations wherever applicable.

Q-1 Write the question number and appropriate **option number only** for each question. [8]

(a) _____ topology on $\mathbb{R}$ is not Hausdorfff.
(i) Cofinite (ii) Usual (iii) Lower limit (iv) Discrete

(b) _____ $\subset \mathbb{R}$ as well as its complement has empty interior.
(i) $\mathbb{N}$ (ii) $\mathbb{Z}$ (iii) $\mathbb{Q}$ (iv) all of these

(c) A _____ sequence is _____ but not conversely.
(i) convergent, bounded (iii) unbounded, Cauchy
(ii) bounded, Cauchy (iv) Cauchy, Convergent

(d) Every continuous function f : _____ $\to \mathbb{R}$ is uniformly continuous.
(i) $[0,1)$ (ii) $(0,1]$ (iii) $(0,1)$ (iv) $[0,1]$

(e) Range of a _____ sequence need not be compact.
(i) convergent (ii) constant (iii) Cauchy (iv) All of these

(f) Every sequence in _____ has a convergent subsequence.
(i) $(0,1)$ (ii) $[0,1]$ (iii) $\mathbb{Q}$ (iv) $\mathbb{N}$

(g) A _____ space is separable.
(i) metric (ii) second countable (iii) discrete (iv) cocountable

(h) _____ is second countable but not compact.
(i) A discrete space (ii) A metric space (iii) $[0,1]$ (iv) $\mathbb{R}$

Q-2 Attempt *Any Seven* of the following: [14]

(a) Is $(\pi, 7)$ open in the topology on $\mathbb{R}$ generated by the following open base?

$$\{[a,b) : a \in \mathbb{Q}, b \in \mathbb{N}, a < b\}?$$

(b) Find the closure of $\mathbb{Q}$ in the cocountable topology on $\mathbb{R}$.

(c) Define a *metric*. Give an example of a topological space which is not metrizable.

(d) Define the *product topology* on $\mathbb{R}^2$ and give an example of an open subset of $\mathbb{R}^2$.

(e) Define *compact* space and show that a compact discrete space is finite.

(f) Define the term *diameter of a subset of a metric space*. Find the diameter of $\{n + \frac{1}{n} : n \in \mathbb{N}\}$ in $\mathbb{R}$ with the usual metric.

(g) Define a T_3-*space* and show that a discrete space is T_3.

(h) Define the term *a second countable topological space* and show that $\mathbb{R}$ with the usual topology is separable.

(i) Show that a finite topological space is second countable.

Q-3 (j) Define the *usual and the cofinite topologies* on $\mathbb{R}$ and prove that the cofinite topology is weaker than the usual topology. [6]

(k) Define *limit point of a subset* of a topological space. For a subset A of a topological space X, show that $\overline{A} = A \cup A'$. [6]

OR

(k) Define a T_2-*space* and show that a subspace of a T_2-space T_2. [6]

Q-4 (l) Show that convergent sequences in a T_2-space have unique limits. But the result need not hold in a T_1-space. [6]

(m) Show that $A \subset \mathbb{R}$ is bounded if and only if it is totally bounded. [6]

OR

(m) For a product space $X = \prod_{i=1}^{n} X_i$, show that X is T_2 if and only if each X_i is T_2. [6]

Q-5 (n) Define *an open cover of a topological space*. Show that a compact subset of a T_2-space is closed. [6]

(o) Show that a continuous image of a compact space is compact. [6]

OR

(o) Show that a compact metric space is bounded but the converse does not hold. [6]

Q-6 (p) Show that a topological space X is T_4 if and only if for every open set $G \subset X$ and a closed set $E \subset G$, there exists an open set $H \subset X$ such that $E \subset H \subset \overline{H} \subset G$. [6]

(q) Show that a normal space is regular. [6]

OR

(q) Show that $\mathbb{R}$ with cofinite topology is not second countable. [6]

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