SC

SEAT No._______

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[41]

Sardar Patel University
Mathematics
M.Sc. Semester I
Tuesday, 10 April 2018
10.00 a.m. to 01.00 p.m.
PS01CMTH01 - Complex Analysis I

Maximum Marks: 70

Q.1 Fill in the blanks. [8]

(1) The minimum value of $\{|3e^{i\varphi} - 2e^{i\theta}| : \theta, \varphi \in \mathbb{R}\}$ is _____

(a) 1 (b) 2 (c) 3 (d) 5

(2) $\mathrm{Arg}(2+2i) + \mathrm{Arg}(-2-2i) =$ _____

(a) 0 (b) π (c) 2π (d) $\{2n\pi : n \in \mathbb{Z}\}$

(3) Suppose that $\lim_{z\to z_0} |f(z)| = 0$. Then $\lim_{z\to z_0} f(z)$ _____

(a) may not exist (b) 0 (c) z_0 (d) $|z_0|$

(4) Which of the following functions is not an entire function?

(a) e^{z^2} (b) e^{z^3} (c) e^{z^4} (d) $e^{\frac{1}{z^2}}$

(5) $\int_{|z|=3} \frac{(4-z)^2}{z} dz =$ _____

(a) $32\pi i$ (b) $16\pi i$ (c) $8\pi i$ (d) none of these

(6) Which of the following is a bounded function on $\mathbb{C}$?

(a) $|z|$ (b) $|z|^2$ (c) $|\sin z|$ (d) none of these

(7) The residue of $\frac{\sin^2 z}{z^2}$ at 0 is

(a) 0 (b) $2\pi i$ (c) $\frac{\pi i}{2}$ (d) πi

(8) 0 is _____ of $\frac{\sinh z}{z}$.

(a) Essential singularity (c) removable singularity
(b) pole (d) non isolated singularity

Q.2 Attempt any *Seven*. [14]

(a) Show that a complex number z is nonnegative if and only if $|z| = z$.
(b) Is it true that $\mathrm{Arg}(\frac{z}{w}) = \mathrm{Arg}(z) - \mathrm{Arg}(w)$? Why?
(c) Show that $\lim_{z\to 0} \frac{\overline{z}}{z}$ does not exist.
(d) Find the real and imaginary parts of $z^2 e^z$.
(e) State Principle of deformation of paths.

(P. T. O.)

(f) Evaluate $\int_{|z|=1} \frac{\cosh z}{z^5} dz$.

(g) Let D be a domain and $f : D \to \mathbb{C}$. If F and G are antiderivatives of f on D, then show that $G = F + c$ for some constant c.

(h) Find the Laurent series of $\frac{1}{(z-i)(z-2i)}$ in the region $1 < |z| < 2$.

(i) Find the fixed points of $w(z) = \frac{2z+1}{z+2}$.

Q.3

(a) Let $n \in \mathbb{N} \setminus \{1\}$. Find all the solutions of the equation $z^{12} = 12$. Also, find the sum and product of all these solutions. [6]

(b) Define argument and principal argument of a nonzero complex number. Let z_1, z_2 be nonzero complex numbers. Is it true that $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$? Justify. [6]

OR

(b) Let $z_1, z_2, \ldots, z_n \in \mathbb{C}$. Show that $\left|\sum_{k=1}^{n} (-1)^k z_k\right| \leq \sum_{k=1}^{n} |z_k|$. State the results you use.

Q.4

(c) Let D be a domain, and let $f : D \to \mathbb{C}$ be analytic. If $f'(z) = 0$ for all $z \in D$, then show that f is a constant map. Is the same true if D is not connected? Why? [6]

(d) Let $f = u + iv$ be defined in a neighbourhood of $z_0 = x_0 + iy_0$. If u_x, u_y, v_x and v_y are continuous in a neighbourhood of (x_0, y_0) and if the Cauchy - Riemann equations are satisfied at (x_0, y_0), then show that f is differentiable at z_0. [6]

OR

(d) Define a harmonic conjugate of a harmonic function on a domain. Show that $e^{x^2-y^2}\cos(2xy)$ is harmonic on $\mathbb{R}^2$. Find an analytic function having the imaginary part $v(x, y) = e^{2x}\sin 2y - y$. [6]

Q.5

(e) If P is a polynomial of degree $n \geq 1$, then show that there is $w \in \mathbb{C}$ such that $P(w) = 0$. [6]

(f) Let $C : z(t)$, $a \leq t \leq b$, and let f be piecewise continuous on C. Define $\int_C f(z)dz$. Let $C' : w(t)$, $-b \leq t \leq -a$, where $w(t) = z(-t)$. Is there any relation between $\int_C f(z)dz$ and $\int_{C'} f(z)dz$? Justify. [6]

OR

(f) State Maximum Modulus Principle. Suppose that a function f is continuous on a closed bounded region R and that it is analytic and not constant in the interior of R. Show that the maximum value of $|f|$ in R occurs somewhere on the boundary of R and never in the interior. [6]

Q.6

(g) Let $f(z) = \exp(z)$ for all $z \in \mathbb{C}$. Prove that $f(z) = \sum_{n=0}^{\infty} \frac{z^n}{n!}$ for all $z \in \mathbb{C}$. State the result you use. [6]

(h) State Cauchy's Residue Theorem. Hence evaluate $\int_{|z|=3} \frac{z+1}{z^2-2z} dz$. [6]

OR [6]

(h) Evaluate $\int_0^{\infty} \frac{\sin 2x}{2x} dx$. [6]