Sc

SEAT No.________

[80]

No of printed pages: 2

Sardar Patel University

Mathematics

M.Sc. Semester I

Wednesday, 01 November 2017

2.00 p.m. to 5.00 p.m.

PS01CMTH01 - Complex Analysis I

Maximum Marks: 70

Q.1 Fill in the blanks. [8]

(1) The minimum value of $\{|6 - 2e^{i\theta}| : \theta \in \mathbb{R}\}$ is _____

(a) 2 (b) 4 (c) 6 (d) 8

(2) $\text{Arg}(1+i) + \text{Arg}(-1-i) =$ _____

(a) 0 (b) π (c) 2π (d) $\{2n\pi : n \in \mathbb{Z}\}$

(3) Suppose that $\lim_{z\to z_0} |f(z)| = |w_0|$. Then $\lim_{z\to z_0} f(z)$ _____

(a) may not exist (b) w_0 (c) $|w_0|$ (d) none of these

(4) Which of the following functions is not an entire function?

(a) e^{-z} (b) e^{-z^2} (c) e^{-z^3} (d) $e^{\frac{1}{z}}$

(5) $\int_{|z|=3} \frac{z^2}{z-4} dz =$ _____

(a) $32\pi i$ (b) $16\pi i$ (c) $8\pi i$ (d) none of these

(6) Which of the following is a bounded function on $\mathbb{C}$?

(a) z (b) z^2 (c) $\sin z$ (d) none of these

(7) The residue of $\frac{1-\cos z}{z^2}$ at 0 is

(a) 0 (b) $2\pi i$ (c) $\frac{\pi i}{2}$ (d) πi

(8) 0 is _____ of $\cosh(\frac{1}{z})$.

(a) Essential singularity (c) removable singularity
(b) pole (d) non isolated singularity

Q.2 Attempt any *Seven*. [14]

(a) Show that a complex number z is real if and only if $\overline{z} = z$.
(b) Is it true that $\text{Arg}(zw) = \text{Arg}(z) + \text{Arg}(w)$? Why?
(c) Show that $\lim_{z\to 0} \frac{\overline{z}^2}{z^2}$ does not exist.
(d) Find the real and imaginary parts of e^{-z^3}.
(e) State Morera's theorem.
(f) Evaluate $\int_{|z|=1} \frac{(z-1)^2}{z^3} dz$.
(g) Let D be a domain and $f : D \to \mathbb{C}$. Define antiderivative of f on D. If F and G are antiderivatives of f on D, then show that $F - G$ is a constant map.

(h) Find the Laurent series of $\frac{1}{(z-1)(z-2)}$ in the region $1 < |z| < 2$.

(i) Find the inverse of a bilinear transformation $w(z) = \frac{3z+4}{4z+3}$.

Q.3

(a) Let $n \in \mathbb{N} \setminus \{1\}$. Find all the solutions of the equation $z^n = 3$. Also, find the sum and product of all these solutions. [6]

(b) Define argument and principal argument of a nonzero complex number. Let $z_1, z_2, \ldots, z_n$ be nonzero complex numbers. Is it true that $\arg(z_1 z_2 \cdots z_n) = \arg(z_1) + \arg(z_2) + \cdots + \arg(z_n)$? Justify. [6]

OR

(b) If $a, b \in \mathbb{C}$, then show that $|a - b| \leq |a| + |b|$. Is it true that $||a| - |b|| \leq |a + b|$? Justify. [6]

Q.4

(c) Let D be a domain, and let $f : D \to \mathbb{C}$ be analytic. Show that f is a constant map if f satisfies any of the following conditions. (a) $\overline{f}$ is analytic on D. (b) f is real valued. (c) $|f|$ is a constant map. [6]

(d) Let $f = u + iv$ be defined in a neighbourhood of $z_0 = x_0 + iy_0$. If f is differentiable at z_0, then show that $u_x(x_0, y_0) = v_y(x_0, y_0)$ and $u_y(x_0, y_0) = -v_x(x_0, y_0)$. Is the converse true? Why? [6]

OR

(d) Define a harmonic conjugate of a harmonic function on a domain. Show that harmonic conjugate of a harmonic function need not be unique. Also, show that if a function is harmonic conjugate of itself, then it is a constant map. [6]

Q.5

(e) Let f be an entire function. If f is nonconstant, then show that f is not bounded. Deduce that z^4 is an unbounded function. [6]

(f) Let $C : z(t)$, $a \leq t \leq b$, and let f be piecewise continuous on C. Define $\int_C f(z)dz$. In usual notations prove that $\int_C f(z)dz + \int_{-C} f(z)dz = 0$. [6]

OR

(f) State Maximum Modulus Principle. Suppose that a function f is continuous on a closed bounded region R and that it is analytic and not constant in the interior of R. Show that the maximum value of $|f|$ in R occurs somewhere on the boundary of R and never in the interior. [6]

Q.6

(g) If f is analytic on $N(z_0, R)$, then show that f has a power series representation $f(z) = \sum_{n=0}^{\infty} \frac{f^{(n)}(z_0)}{n!}(z - z_0)^n$ for all $z \in N(z_0, R)$. [6]

(h) State Cauchy's Residue Theorem. Hence evaluate $\int_C \frac{z+2}{z^2-z}dz$, where C is a positively oriented circle $|z| = 2$. [6] [6]

OR

(h) Evaluate $\int_0^{2\pi} \frac{\cos^2 3\theta}{5+4\cos 2\theta}d\theta$. [6]

ႹႹႹႹႹႹႹႹႹ

SC

[160] SEAT No.________ No. of printed pages: 2

SARDAR PATEL UNIVERSITY

M.Sc. (Mathematics) Semester - I Examination

Tuesday, 07th November, 2017

PS01CMTH02, Topology-I

Time: 02:00 p.m. to 05:00 p.m. Maximum marks: 70

Note: Figures to the right indicate full marks of the respective questions.
Assume standard notations wherever applicable.

Q-1 Write the question number and appropriate **option number only** for each question. [8]

(a) _____ topology on $\mathbb{R}$ is the smallest T_1-topology.
(i) Cofinite (ii) Usual (iii) Lower limit (iv) Discrete

(b) No subset of $\mathbb{R}$ with _____ topology has a limit point.
(i) cofinite (ii) usual (iii) lower limit (iv) discrete

(c) A polynomial of degree _____ defines a uniformly continuous function on $\mathbb{R}$.
(i) 1 (ii) 2 (iii) 3 (iv) 4

(d) Diameter of $\mathbb{R}$ with the metric $d(x, y) = \frac{|x-y|}{1+|x-y|}$, $(x, y \in \mathbb{R})$, is _____.
(i) 1 (ii) 2 (iii) 3 (iv) ∞

(e) _____ is a dense as well as a subset of first category in $\mathbb{R}$.
(i) $\mathbb{N}$ (ii) $\mathbb{Z}$ (iii) $\mathbb{Q}$ (iv) $\mathbb{R}$

(f) _____ subset of a metric space need not be closed.
(i) complete (ii) compact (iii) countable (iv) derived set of a subset

(g) _____ topology makes every set a compact topological space.
(i) usual (ii) cofinite (iii) discrete (iv) cocountable

(h) _____ topology on $\mathbb{R}$ is T_3 but not regular.
(i) usual (ii) lower limit (iii) discrete (iv) indiscrete

Q-2 Attempt *Any Seven* of the following: [14]

(a) Show that $\{(a, \infty) : a \in \mathbb{R}\}$ is a base for a topology on $\mathbb{R}$.

(b) Find the closure of $\{1, 2\}$ in cofinite topology on $\mathbb{R}$.

(c) Give a topology on $\{1, 2, 3, 4, 5, 6\}$ making it a T_2-space.

(d) Define the term *product topology*. Give an example of an open subset of $\mathbb{R} \times \mathbb{R}$.

(e) Define the term *metric on a set* and give an example of a metric space.

(f) Define the term *diameter of a subset of a metric space*. Find the diameter of $\{2 + \frac{1}{n} : n \in \mathbb{N}\} \cup \{3 - \frac{1}{n} : n \in \mathbb{N}\}$ in $\mathbb{R}$ with the usual metric.

(g) Define the term *open cover*. Give an open cover of $\mathbb{R}$ with the usual topology.

(h) Define the term *a separable topological space* and show that $\mathbb{R}$ with the usual topology is separable.

(i) Define and give an example of a *disconnected space*.

(P.T.O.)

(1)

Q-3 (j) Define the *usual and the lower limit topologies* on $\mathbb{R}$ and prove that the lower limit topology is finer than the usual topology. [6]

(k) Define *interior and closure of a subset* of a topological space. Show that a subset A of a topological space X is open if and only if $A = A^{\circ}$. [6]

OR

(k) State and prove Pasting Lemma. [6]

Q-4 (l) Show that the product topology is the weakest topology on the product space with respect to which the projections are continuous. [6]

(m) State and prove Cantor's Intersection Theorem [6]

OR

(m) Consider the product space $X = \prod_{i=1}^{n} X_i$. Then show that X is T_2 if and only if each X_i is T_2. [6]

Q-5 (n) Let $(X, \mathscr{T})$ be a topological space and $(Y, \mathscr{T}_Y)$ be its subspace. Show that Y is compact in Y if and only if Y is compact in X. [6]

(o) Show that a compact subset of a T_2-space is closed. [6]

OR

(o) Show that a compact metric space is totally bounded but the converse does not hold. [6]

Q-6 (p) Show that a topological space X is T_3 if and only if for every open set $G \subset X$ and a point $x \in G$, there exists an open set $H \subset X$ such that $x \in H \subset \overline{H} \subset G$. [6]

(q) Show that a metric space is T_4. [6]

OR

(q) Show that a continuous image of a connected space is connected. [6]

SEAT No.________

SC

No. of Printed Pages : 02

[73] Sardar Patel University, Department of Mathematics

M.Sc. (Mathematics) External Examination 2017;

Code:- PS01CMTH03 : Subject :- Functions of Several Real Variables;

Date: 03-11-2017, Friday; Time- 02.00 pm to 05.00 pm ; Max. Marks 70

Note: Notations and Terminologies are standard.

Q.1 Choose correct option from given four choices. [08]

(i) Let $x = (1, -1, 2)$ and $y = (1, 1, -2)$. Then $\|x + y\| =$

(a) 0 (b) 2 (c) 4 (d) 6

(ii) Let $x, y \in \mathbb{R}^n$. Then $\|x - y\| \leq$

(a) $\|x\|$ (b) $\|y\|$ (c) $\|x\| + \|y\|$ (d) $\|x\| - \|y\|$

(iii) Let $T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ be a linear map.

(a) If T is norm preserving, then T is inner product preserving

(b) If T is inner product preserving, then T is norm preserving

(c) If T is norm preserving, then T is angle preserving

(d) If T is angle preserving, then T is norm preserving

(iv) Let $f : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ be differentiable at $a \in \mathbb{R}^n$. Then its Jacobian matrix $f'(a)$ is

(a) $n \times m$ (b) $m \times mn$ (c) $mn \times m$ (d) none

(v) Let $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ such that $D_x f(a)$ exists for all $x \in \mathbb{R}^n$. Then

(a) f is continuous at a

(b) $D_j f(a)$ exists for all $1 \leq j \leq n$

(c) f is continuously differentiable at a

(d) f is differentiable at a

(vi) Let $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ be defined as $f(x) = |x_1|$. Then

(a) f is differentiable at 0

(b) $D_x f(0)$ exists for all $x \in \mathbb{R}^n$

(c) f is continuous at 0

(d) $D_j f(0)$ exists for all $1 \leq j \leq n$

(vii) Let $S \in \mathcal{T}^3(V)$ and $T \in \mathcal{T}^5(V)$. Then $S \otimes T$ belongs to

(a) $\mathcal{T}^2(V)$ (b) $\mathcal{T}^3(V)$ (c) $\mathcal{T}^5(V)$ (d) $\mathcal{T}^8(V)$

(viii) Let π_1 and π_2 be projection maps on $\mathbb{R}^2$. Then $\pi_1 \wedge \pi_2 =$

(a) $\pi_1 \otimes \pi_2 - \pi_2 \otimes \pi_1$ (b) $\pi_1 \otimes \pi_2 + \pi_2 \otimes \pi_1$ (c) $\pi_1 \otimes \pi_2$ (d) $\pi_2 \otimes \pi_1$

Q.2 Attempt any seven. [14]

(i) Prove that $\|x + y\| \leq \|x\| + \|y\|$ $(x, y \in \mathbb{R}^n)$.

(ii) Let $f, g : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ be continuous at a. Prove that fg is also continuous at a.

(iii) Prove that every differentiable function is continuous.

(iv) Let $a = (1, 0)$ and $f : \mathbb{R}^2 \to \mathbb{R}$ be defined as $f(x) = e^{x_1}$. Find $Df(a)$.

(v) Let $f : \mathbb{R}^n \to \mathbb{R}^m$ be differentiable at a. Prove that each $f^i : \mathbb{R}^n \to \mathbb{R}$ is differentiable at a.

(vi) Let $s \in \mathbb{R}$, $a, x \in \mathbb{R}^n$ and $f : \mathbb{R}^n \to \mathbb{R}$ be differentiable at a. Prove that $D_{sx} f(a) = s D_x f(a)$.

(vii) Give example of a function $f : \mathbb{R}^2 \to \mathbb{R}$ such that $D_x f(0)$ exist but $Df(a)$ does not exist.

(viii) Define an inner product on a vector space V.

(ix) Let $T \in \mathcal{T}^2(\mathbb{R}^2)$ be defined as $T(x, y) = x_1 y_1$. Find Alt(T).

(Continue on page-2)

Q.3

(a) Define the inner product on $\mathbb{R}^n$. Then state and prove the polarization identity. [6]

(b) Let $A \subset \mathbb{R}^n$, let $f : A \longrightarrow \mathbb{R}$ be a bounded function, and let $a \in A$. Then prove that f is continuous at a iff $o(f;a) = 0$. [6]

OR

(b) Let $A \subset \mathbb{R}^n$ be closed, let $f : A \longrightarrow \mathbb{R}$ be a bounded function, and let $\varepsilon > 0$. Then prove that the set $B = \{x \in A : o(f;x) \geq \varepsilon\}$ is closed in $\mathbb{R}^n$. [6]

Q.4

(a) State and prove the chain rule. [6]

(b) Let $f, g : \mathbb{R}^n \longrightarrow \mathbb{R}$ be differentiable at a and $g(a) \neq 0$. Then prove that $\frac{f}{g}$ is differentiable at a. [6]

OR

(b) Define $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ as $f(x) = |x_1|\sqrt{|x_2|}\,(x \in \mathbb{R}^2)$. Does $Df(0)$ exist? If not, then why? If, yes, then find it. [6]

Q.5

(a) Let $f : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ be differentiable at $a \in \mathbb{R}^n$, let $1 \leq i \leq m$, and let $1 \leq j \leq n$. Then prove that the $(j,i)^{th}$-entry of the Jacobian matrix $f'(a)$ is exactly the j^{th}-entry of the Jacobian matrix $f^{i'}(a)$. [6]

(b) Let $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ be differentiable at $a \in \mathbb{R}^n$. Then prove that $D_x f(a) = Df(a)(x)$ and $D_{x+y} f(a) = D_x f(a) + D_y f(a)$. [6]

OR

(b) Define $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ by

$$f(x) = \begin{cases} \|x\|^2 \sin(\frac{1}{\|x\|}) & (\text{if } x \neq 0) \\ 0 & (\text{if } x = 0) \end{cases}$$

Is f differentiable at 0? Justify your answer. [6]

Q.6

(a) Let $S \in \mathcal{T}^k(V)$ such that $\text{Alt}(S) = 0$ and $T \in \mathcal{T}^\ell(V)$. Prove that $\text{Alt}(S \otimes T) = 0$. [6]

(b) Let V be a vector space with dimension n and $k \in \mathbb{N}$ such that $k < n$. Prove that the dimension of $\Lambda^k(V)$ is $\frac{n!}{k!(n-k)!}$. [6]

OR

(b) Let $f : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ be differentiable. Define the map $\widetilde{f}_{k*} : \Delta_{kF}(\mathbb{R}^m) \to \Delta_{kF}(\mathbb{R}^n)$ and prove that it is a linear map. [6]

THE END

SC

(183) SEAT No.________ No. of printed pages: 2

SARDAR PATEL UNIVERSITY

M.Sc. (Mathematics) Semester - I Examination

Thursday, 9^{th} November, 2017

PS01CMTH04, Linear Algebra

Time: 02:00 p.m. to 05:00 p.m. Maximum marks: 70

Note: 1. Figures to the right indicate marks of the respective question.
2. Assume standard notations wherever applicable.

Q-1 Choose the most appropriate option for each of the following questions: [8]

1. The dimension of the vector space $\mathbb{C}^3$ over $\mathbb{R}$ is ____.
   (a) 6 (b) 4 (c) 3 (d) infinite
2. Let W be a subspace of a vector space V over F. If $\dim V = 5$ and $\dim W = 2$, then $\dim W^0 =$____.
   (a) 5 (b) 3 (c) 2 (d) 0
3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined as $T(x_1, x_2, x_3) = (x_2, x_3, 0)$. Then the minimal polynomial of T is ____.
   (a) $p(x) = x$ (b) $p(x) = x^2$ (c) $p(x) = x^3$ (d) $p(x) = 0$
4. Let V be any vector space and $S \in A(V)$ be right invertible. Then ____.
   (a) S is singular (b) S is regular (c) S is onto (d) nothing can be said
5. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be linear and nilpotent with invariants 1, 1. Then T is ____.
   (a) regular (b) singular (c) I (d) 0
6. Let V vector space over F and $T \in A(V)$ be nilpotent. Then $I + T$ is ____.
   (a) nilpotent (b) singular (c) regular (d) none of these
7. Let $A \in M_n(F)$ be nilpotent. Then $\mathrm{tr}(A) =$____.
   (a) 1 (b) 0 (c) -1 (d) n
8. Let F be a field, $A \in M_n(F)$ and $\alpha \in F$, $\alpha \neq 0$. Then $\det(\alpha A) =$____.
   (a) $\alpha^n \det(A)$ (b) $\alpha \det(A)$ (c) $\frac{\det(A)}{\alpha}$ (d) $\frac{\det(A)}{\alpha^n}$

Q-2 Attempt *Any Seven* of the following: [14]

(a) Show that $v_1 = (1, -1, 2)$, $v_2 = (2, 0, -5)$, $v_3 = (8, -2, -11)$ are linearly dependent over $\mathbb{R}$.

(b) Show that if U_1 and U_2 are subspaces of a vector space V over a field F, then $U_1 \cap U_2$ is also a subspace of V.

(c) Let V be a finite dimensional vector space over a field F and $S, T \in A(V)$. If S is regular then show that T and $S^{-1}TS$ satisfy the same polynomial $p(x) \in F[x]$.

(d) Define $T : \mathbb{R}^3 \to \mathbb{R}^3$ by $T(x, y, z) = (y + z, x + z, x + y)$, $(x, y, z) \in \mathbb{R}^3$. Find the matrix of T with respect to standard basis of $\mathbb{R}^3$.

(e) Define nilpotent linear transformation on a vector space. Show that $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (0, x, y)$ is nilpotent.

(f) Define invariant subspace of a vector space under a linear transformation. Show that if $T : \mathbb{R}^2 \to \mathbb{R}^2$ is defined by $T(x, y) = (x + y, x + y)$, for all $(x, y) \in \mathbb{R}^2$, then $W = \{(x, x) \mid x \in \mathbb{R}\}$ is invariant under T.

(g) Let F be a field and $A \in M_n(F)$ be regular. Then show that $\det(A^{-1}) = \frac{1}{\det(A)}$.

(h) Find the symmetric matrix associated to the quadratic form:
$x_1^2 + x_2^2 + x_3^2 + 2x_1x_2 + 2x_1x_3 + 2x_2x_3 = 0$.

(i) For $A, B \in M_n(\mathbb{R})$, show that $\mathrm{tr}(A+B) = \mathrm{tr}(A) + \mathrm{tr}(B)$.

Q-3 (a) Let V be a finite-dimensional vector space over a field F and W be a subspace of V. Show that W is also finite-dimensional and in fact $\dim V/W = \dim V - \dim W$. [6]

(b) Show that internal direct sum of vector spaces is isomorphic to external direct sum. [6]

OR

(b) Let U and V be vector spaces over F and T be a homomorphism of U onto V with kernel W. Show that V is isomorphic to U/W. [6]

Q-4 (a) Let V be a vector space over F and $T \in A(V)$. Show that T is regular if and only if the constant term of the minimal polynomial for T is non-zero. [6]

(b) Let V be a finite dimensional vector space over F and $S, T \in A(V)$. Show that $r(TS) \leq r(T)$ and $r(ST) \leq r(T)$. Further if S is regular then show that $r(ST) = r(TS) = r(T)$. [6]

OR

(b) If V is an n-dimensional vector space over F, then show that $A(V)$ and $M_n(F)$ are isomorphic as algebras over F. [6]

Q-5 (a) Let V be a finite dimensional vector space over F and $T \in A(V)$ be nilpotent. Prove that the invariants of T are unique. [6]

(b) Let V be a finite dimensional vector space over F and $T \in A(V)$ be such that all the characteristic roots of T are in F. Show that there exists a basis of V with respect to which the matrix of T is upper triangular. [6]

OR

(b) Let V be a finite dimensional vector space over F, $T \in A(V)$ and W be a subspace of V invariant under T. Let $\overline{T} = V/W \to V/W$ be defined by $\overline{T}(v + W) = Tv + W$. Show that $\overline{T} \in A(V/W)$ and $\overline{T}$ satisfies every polynomial in $F[x]$ satisfied by T. [6]

Q-6 (a) For $A, B \in M_n(F)$, show that $\det(AB) = \det(A)\det(B)$. [6]

(b) Show that if any two rows of $A \in M_n(F)$ are equal then $\det(A) = 0$. Is the converse true? Justify your answer. [6]

OR

(b) i. State and prove Cramer's rule. [4]

ii. Find the symmetric matrix associated with the following quadratic form: [2]
$9x_1^2 - x_2^2 + 4x_3^2 + 6x_1x_2 - 8x_1x_3 + 2x_2x_3$.

\# \# \# \# \# \# \# \# \# \# \# \# \# \# \# \# \# \#

— x —

(2)

Sc

(172) SEAT No.________ No of printed pages: 2

Sardar Patel University

M.Sc. (Sem-I), PS01CMTH05, Methods of Differential Equations;
Saturday, 11^{th} November, 2017; 02.00 p.m. to 05.00 p.m.

Maximum Marks: 70

Note: (i) Notations and terminologies are standard; (ii) Figures to the right indicate marks.

Q.1 Answer the following. [8]

1. The order of differential equation $y'' - x^3(y')^2 = 0$ is
   (A) 1 (B) 2 (C) 3 (D) 0
2. The set of singular points of $xy'' + xy' + y = 0$ is
   (A) $\{0\}$ (B) φ (C) $\{1\}$ (D) none of these
3. $\int_{-1}^{1} J_1(x)dx =$
   (A) $\sqrt{\pi}$ (B) 0 (C) -1 (D) none of these
4. $\int_{-1}^{1} x^2 P_3(x)dx =$
   (A) 0 (B) $\frac{2}{15}$ (C) $\frac{4}{15}$ (D) none of these
5. Which of the following is an integrating factor of $ydx - xdy$?
   (A) $\frac{1}{y}$ (B) $\frac{1}{x}$ (C) $\frac{1}{y^2}$ (D) none of these
6. Which one is homogeneous Pfaffian differential equation?
   (A) $xdx + ydy + zydz = 0$
   (B) $(x^2+1)dx + (y^2+1)dy + (z^2+1)dz = 0$
   (C) $xydx + yzdy + zxdz = 0$
   (D) none of these
7. $F(1, \frac{1}{5}; \frac{1}{5}; -2) =$
   (A) 1 (B) 3 (C) -1 (D) none of these
8. $F(\alpha, \beta; \gamma; 0)$ equals
   (A) -1 (B) 0 (C) $\frac{\alpha\beta}{\gamma}$ (D) 1

Q.2 Attempt any *seven*: [14]

(a) Find the interval of convergence of $\sum_{n=1}^{\infty} \frac{1}{(n!)^2} x^n$.
(b) Verify the analyticity of $xy'' - (\cos x - 1)y' + xy = 0$ at 0.
(c) Show that $J_n(x) = (-1)^n J_n(-x)$ where $n \in \mathbb{Z}$.
(d) Show that $\Gamma(x+1) = x\Gamma(x)$, $x > 0$.
(e) State Fourier-Legendre expansion theorem.
(f) State Picard's theorem.
(g) Find a partial differential equation by eliminating a and b from $z = ax + by$.
(h) Show that $F(\alpha, \beta; \beta; x) = (1-x)^{-\alpha}$.
(i) Find radius of convergence of Gauss's hypergeometric series.

(1) (P.T.O)

Q.3

(a) Solve: $x^2y'' + \frac{3}{2}xy' - \frac{1}{2}(x+1)y = 0$ near origin. [6]

(b) Classify the singularities of $(x-2)(x-1)y'' + \sin(x-2)y' + e^x(x-1)y = 0$. [6]

OR

(b) Find the general solution of $y'' - xy' - y = 0$ in terms of power series in x.

Q.4

(a) State and prove Rodrigue's formula. [6]

(b) Prove: $\frac{d}{dx}[x^\alpha J_\alpha(x)] = x^\alpha J_{\alpha-1}(x)$ [6]

OR

(b) Show that between two consecutive positive roots of J_1 there is unique root of J_0.

Q.5

(a) Solve $y' = y,\ y(0) = 1$ using Picard's method of successive approximations. [6]

(b) Find a necessary and sufficient condition that there exists between two functions $u(x,y)$ and $v(x,y)$, a relation $F(u,v) = 0$ not involving x or y explicitly. [6]

OR

(b) Verify that the differential equation $(y^2 + yz)dx + (z^2 + zx)dy + (y^2 - xy)dz = 0$ is integrable and find its primitive.

Q.6 [6]

(a) Prove: $P_n(x) = F\left(-n, n+1; 1; \frac{1-x}{2}\right)$.

(b) Solve: $(p^2 + q^2)y = qz$ using Charpit's method [6]

OR

(b) Explain Jacobi's method.

*****

— x —

2

SC

[A-61] SEAT No.________ No. of printed pages: 2

Sardar Patel University

M.Sc. (Mathematics) Semester - I Examination

Tuesday, 07^{th} November, 2017

PS01CMTH07, Topology-I

Time: 02:00 p.m. to 05:00 p.m. Maximum marks: 70

Note: Figures to the right indicate full marks of the respective questions.
Assume standard notations wherever applicable.

Q-1 Write the question number and appropriate **option number only** for each question. [8]

(a) _____ topology on $\mathbb{R}$ is the smallest T_1-topology.
(i) Cofinite (ii) Usual (iii) Lower limit (iv) Discrete

(b) No subset of $\mathbb{R}$ with _____ topology has a limit point.
(i) cofinite (ii) usual (iii) lower limit (iv) discrete

(c) A polynomial of degree _____ defines a uniformly continuous function on $\mathbb{R}$.
(i) 1 (ii) 2 (iii) 3 (iv) 4

(d) Every subset of the _____ space is closed.
(i) discrete (ii) indiscrete (iii) cofinite (iv) cocountable

(e) If a set has no limit point, then it is _____.
(i) open (ii) dense (iii) closed (iv) bounded

(f) _____ subset of a metric space need not be closed.
(i) complete (ii) compact (iii) countable (iv) derived set of a subset

(g) _____ topology makes every set a compact topological space.
(i) usual (ii) cofinite (iii) discrete (iv) cocountable

(h) _____ topology on $\mathbb{R}$ is T_3 but not regular.
(i) usual (ii) lower limit (iii) discrete (iv) indiscrete

Q-2 Attempt *Any Seven* of the following: [14]

(a) Define *cofinite topology.*

(b) Define and give an example of a *closed set.*

(c) Give a topology on $\{1, 2, 3, 4, 5, 6\}$ making it a T_2-space.

(d) Define and give an example of a *uniformly continuous function.*

(e) Define the term *metric on a set* and give an example of a metric space.

(f) Define and give example of *convergent sequence.*

(g) Define the term *open cover.* Give an open cover of $\mathbb{R}$ with the usual topology.

(h) Define the term *a separable topological space* and show that $\mathbb{R}$ with the usual topology is separable.

(i) Define and give an example of a T_2*-space.*

(P.T.O.)

Q-3 (j) Define the *usual and the lower limit topologies* on $\mathbb{R}$ and prove that the lower limit topology is finer than the usual topology. [6]

(k) Define *interior and closure of a subset* of a topological space. Show that a subset A of a topological space X is open if and only if $A = A^\circ$. [6]

OR

(k) State and prove Pasting Lemma. [6]

Q-4 (l) Define a *bounded subset* of a metric space, a *totally bounded* subset of a metric space. Show that a totally bounded subset is bounded but the converse does not hold. [6]

(m) Define a T_1*-space*. Show that a metric space is T_2 and also show that a T_1-space need not be T_2. [6]

OR

(m) Consider the product space $X = \prod_{i=1}^{n} X_i$. Then show that X is T_2 if and only if each X_i is T_2. [6]

Q-5 (n) Let $(X, \mathscr{T})$ be a topological space and $(Y, \mathscr{T}_Y)$ be its subspace. Show that Y is compact in Y if and only if Y is compact in X. [6]

(o) Show that a compact subset of a T_2-space is closed. [6]

OR

(o) Show that a compact metric space is totally bounded but the converse does not hold. [6]

Q-6 (p) Show that a topological space X is T_3 if and only if for every open set $G \subset X$ and a point $x \in G$, there exists an open set $H \subset X$ such that $x \in H \subset \overline{H} \subset G$. [6]

(q) Show that a metric space is T_4. [6]

OR

(q) Show that subspace of a T_3-space is T_3. [6]

\#

— x —

(2)

Sc

SEAT No.________

[81]

No of printed pages: 2

Sardar Patel University
Mathematics
M.Sc. Semester I
Wednesday, 01 November 2017
2.00 p.m. to 5.00 p.m.
PS01CMTH21 - Complex Analysis I

Maximum Marks: 70

Q.1 Fill in the blanks. [8]

(1) Let $a \in \mathbb{C}$. Then the minimum value of $\{|z-a|+|z+a| : z \in \mathbb{C}\}$ is _____

(a) 0 (b) $|a|$ (c) $2a$ (d) None of these

(2) $\text{Arg}(i) + \text{Arg}(-i) =$ _____

(a) 0 (b) π (c) 2π (d) $\{2n\pi : n \in \mathbb{Z}\}$

(3) If v and V are harmonic conjugates of a harmonic function u on a domain D, then which of the following is not true?

(a) $v = V$ (b) $v_x + V_y = v_y + V_x$ (c) $v_x = V_x$ (d) $V_{xx} + V_{yy} = 0$

(4) The set of singularity of the function $\cot 2z$ is _____

(a) $\{n\pi : n \in \mathbb{Z}\}$ (b) $\{2n\pi : n \in \mathbb{Z}\}$ (c) $\{\frac{n\pi}{2} : n \in \mathbb{Z}\}$ (d) $\{\frac{n\pi i}{2} : n \in \mathbb{Z}\}$

(5) $\int_{|z|=1} \frac{z}{z-2} dz =$ _____

(a) 0 (b) $4\pi i$ (c) $2\pi i$ (d) $\frac{1}{\pi i}$

(6) Which of the following is a bounded function on $\mathbb{C}$?

(a) $\cos z$ (b) e^{-z} (c) e^{-z^2} (d) none of these

(7) The Taylor series of $\frac{1}{1+z^2}$ about 2 is valid in $N(2, R)$ if $R =$_____

(a) $\sqrt{5}$ (b) $\sqrt{7}$ (c) $\sqrt{11}$ (d) $\sqrt{13}$

(8) The point 0 is a pole of $\frac{\tan z}{z^2}$ of order _____

(a) 1 (b) 2 (c) 3 (d) 4

Q.2 Attempt any ***Seven***. [14]

(a) If $0 \neq \alpha \in \mathbb{C}$ and $\gamma \in \mathbb{R}$, then show that $\overline{\alpha}z + \alpha\overline{z} + \gamma = 0$ represents a line.
(b) If $\lim_{z \to z_0} f(z) = w_0$ and $w_0 \neq 0$, then show that there is $\delta > 0$ such that $|f(z)| > 0$ whenever $0 < |z - z_0| < \delta$.
(c) Let $n \in \mathbb{N} \setminus \{1\}$. Find the sum of all complex numbers satisfying $z^n = 2$.
(d) If $z \in \mathbb{C}$, then show that $\cosh^2 z - \sinh^2 z = 1$.
(e) If f is an entire function, then show that $g(z) = \overline{f(\overline{z})}$ is an entire function.
(f) Let m and n be integers, and let $C : z(\theta) = e^{i\theta}, 0 \leq \theta \leq 2\pi$. Evaluate $\int_C z^n \overline{z}^m dz$.

(g) Let f be the function $f(z) = e^z$ and R the rectangular region $[0,1] \times [0,\pi]$. Find the points in R where $u(x,y) = \text{Re } f(z)$ reaches its maximum and minimum values.

(h) Find the Taylor series of $\frac{1}{z-2}$ about i.

(i) Find the inverse of a bilinear transformation $w(z) = \frac{2z+3}{3z+2}$.

Q.3

(a) Let $f = u+iv$ be defined in a neighbourhood of $z_0 = x_0 + iy_0$. If the functions u_x, u_y, v_x, v_y are continuous in a neighbourhood of (x_0, y_0) and if $u_x(x_0,y_0) = v_y(x_0,y_0)$ and $u_y(x_0,y_0) = -v_x(x_0,y_0)$, then show that f is differentiable at z_0. [6]

(b) Define $\lim_{z \to z_0} f(z) = \infty$. If P is a polynomial of degree $n \geq 1$, then show that $\lim_{z\to\infty} P(z) = \infty$. [6]

OR

(b) Give an example of of a complex function which is differentiable at exactly one point. Show that the map $g \circ f$ is differentiable at z_0 whenever f is differentiable at z_0 and g is differentiable at $f(z_0)$. [6]

Q.4

(c) Let U be an open subset $\mathbb{C}$. Let $f : U \to \mathbb{C}$ be analytic such that $f'(z) = 0$ for all $z \in \mathbb{C}$. Can we conclude that f is a constant map? Why? If not, what condition on U implies that f is a constant map? Justify. [6]

(d) Let $N(z_0, R)$ be the disc of convergence of the power series $S(z) = \sum_{n=0}^{\infty} a_n(z-z_0)^n$. If C is a contour in $N(z_0,R)$ and g is a continuous function C, then show that $\int_C g(z)S(z)dz = \sum_{n=0}^{\infty} a_n \int_C g(z)(z-z_0)^n dz$. State the results you use. [6]

OR

(d) Suppose that v is a harmonic conjugate of u on a domain D. Show that $f = u + iv$ is analytic on D. Find an analytic function f whose real part is $\frac{2xy}{(x^2+y^2)^2}$. [6]

Q.5

(e) If a function f is analytic and nonconstant in a domain D, then show that $|f|$ has no maximum value in D. State the results you use. [6]

(f) Let $C : z(t)$, $a \leq t \leq b$, and let f be piecewise continuous on C. Define $\int_C f(z)dz$. If $f(z) = \pi \exp(\pi \bar{z})$ and C is the boundary of the square with vertices at the points 0, 1, $1+i$ and i, the orientation of C being in the counterclockwise direction, then evaluate $\int_C f(z)dz$. [6]

OR

(f) If $v : \mathbb{R}^2 \to \mathbb{R}$ is a nonconstant harmonic function, then show that v is unbounded. State carefully the results you use. [6]

Q.6

(g) Let z_0 be an isolated singularity of f. Show that z_0 is a pole of f of order m if and only if there is a function φ which is analytic at z_0, $\varphi(z_0) \neq 0$ and $f(z) = \frac{1}{(z-z_0)^m}\varphi(z)$ for all z in some deleted neighborhood of z_0. Also, show that if $m=1$, then $\text{Res}_{z=z_0} f = \lim_{z\to z_0}(z-z_0)f(z)$ and if $m > 1$, then $\text{Res}_{z=z_0} f = \frac{1}{(m-1)!}\frac{d^{m-1}}{dz^{m-1}}[(z-z_0)^m f(z)]|_{z=z_0}$. [6]

(h) State Cauchy's Residue Theorem. Hence evaluate $\int_C \frac{\cot z}{z^4}dz$ and $\int_C \frac{\sinh z}{z^4(1-z^2)}dz$, where C is a positively oriented circle $|z| = \frac{1}{2}$. [6] [6]

OR

(h) State Laurent's Theorem. Find the Laurent series expansion of $\frac{1}{(z-1)(z-3)}$ about i in (all the three) appropriate regions. [6]

♮♮♮♮♮♮♮♮♮

SC

[161] SEAT No.________ No. of printed pages: 2

SARDAR PATEL UNIVERSITY

M.Sc. (Mathematics) Semester - I Examination

Tuesday, 07^{th} November, 2017

PS01CMTH22, Topology-I

Time: 02:00 p.m. to 05:00 p.m. **Maximum marks: 70**

Note: Figures to the right indicate full marks of the respective questions.
Assume standard notations wherever applicable.

Q-1 Write the question number and appropriate **option number only** for each question. [8]

(a) The ____ topology is not weaker than the ____ topology on $\mathbb{R}$.
(i) cofinite, cocountable (iii) cocountable, lower limit
(ii) usual, lower limit (iv) indiscrete, discrete

(b) No subset of $\mathbb{R}$ with the ____ topology has a limit point.
(i) cofinite (ii) usual (iii) lower limit (iv) discrete

(c) A polynomial of degree ____ defines a uniformly continuous function on $\mathbb{R}$.
(i) 1 (ii) 2 (iii) 3 (iv) 4

(d) Diameter of $\mathbb{R}$ with the metric $d(x,y) = \frac{|x-y|}{1+|x-y|}$, $(x, y \in \mathbb{R})$, is ____.
(i) 1 (ii) 2 (iii) 3 (iv) ∞

(e) $\mathbb{R}$ with the ____ topology is disconnected.
(i) cofinite (ii) usual (iii) lower limit (iv) cocountable

(f) ____ is a dense as well as a subset of first category in $\mathbb{R}$.
(i) $\mathbb{N}$ (ii) $\mathbb{Z}$ (iii) $\mathbb{Q}$ (iv) $\mathbb{R}$

(g) The ____ topology makes every set a compact topological space.
(i) usual (ii) cofinite (iii) discrete (iv) cocountable

(h) A ____ space is separable.
(i) compact (ii) Hausdorff (iii) connected (iv) second counrable

Q-2 Attempt *Any Seven* of the following. [14]

(a) Show that $\{(a, \infty) : a \in \mathbb{R}\}$ is a base for a topology on $\mathbb{R}$.

(b) Find the closure of $\mathbb{N}$ in cofinite topology on $\mathbb{R}$.

(c) Show that a constant map is always continuous.

(d) Let (X, d) be a metric space, $A \subset X$ and $x \in X$. If $d(x, A) = 0$, then show that $x \in \overline{A}$

(e) Show that the discrete topology on $\mathbb{R}$ makes it disconnected.

(f) Mention a topology on $\mathbb{N}$ which makes it a compact space.

(g) Show that a compact subset of $\mathbb{R}$ is bounded.

(h) Define the term a *separable topological space* and show that $\mathbb{R}$ with the cocountable topology is not separable.

(i) State Baire's Category theorem.

 (P.T.O.)

Q-3 (j) Let $(X, \mathscr{T})$ be a topological space and $Y \subset X$. Define the *subspace topology on* Y. Show that it is really a topology on Y. [6]

(k) Let $\mathscr{T}, \mathscr{T}'$ be two topologies on a set X generated by the bases $\mathscr{B}, \mathscr{B}'$ respectively. Show that $\mathscr{T}'$ is finer than $\mathscr{T}$ if and only if for every $B \in \mathscr{B}$ and for every $x \in B$, there exists $B' \in \mathscr{B}'$ such that $x \in B' \subset B$. [6]

OR

(k) Define a *closed set* in a topological space and prove that arbitrary intersection of closed sets is closed. Also give an example to show that arbitrary union of closed sets need not be closed. [6]

Q-4 (l) Define a *continuous function* and a *homeomorphism.* Let X, Y be two topological spaces and $f : X \to Y$ be a function. Show that f is continuous if and only if for each $x \in X$ and for each neighbourhood V of $f(x)$, there is a neighbourhood U of x such that $f(U) \subset V$. [6]

(m) (i) Prove that homeomorphic image of a T_2-space is T_2. [6]

(ii) Prove that $\mathbb{R}$ with lower limit topology and $\mathbb{R}$ with the upper limit topology are homeomorphic.

OR

(m) Let $X = \prod_{\alpha \in J} X_\alpha$ be a product space. Show that the product topology is the weakest topology for which each π_β, $(\beta \in J)$, is continuous. [6]

Q-5 (n) Let X be a topological space $\{A_\alpha : \alpha \in J\}$ be a family of connected spaces. If $\cap A_\alpha \neq \emptyset$, then $\cup A_\alpha$ is connected. [6]

(o) Let $(X, \mathscr{T})$ be a topological space and $(Y, \mathscr{T}_Y)$ be its subspace. Show that Y is compact in Y if and only if Y is compact in X. [6]

OR

(o) Show that a topological space X is compact if and only if every family of closed subsets of X with FIP has a nonempty intersection. [6]

Q-6 (p) Define a T_4-*space.* Show that a topological space X is T_4 if and only if for every open set $G \subset X$ and a closed set $E \subset G$, there exists an open set $H \subset X$ such that $E \subset H \subset \overline{H} \subset G$. [6]

(q) Show that a metric space is T_4. [6]

OR

(q) State and prove Cantor's Intersection Theorem. [6]

#########################

— x —

(2)

SC

SEAT No.________ 02

[74] **Sardar Patel University, Department of Mathematics**

M.Sc. (Mathematics) External Examination 2017;

Code:- PS01CMTH23 : Subject :- Functions of Several Real Variables;

Date: 03-11-2017, Friday; Time- 2.00 pm to 5.00 pm ; Max. Marks 70

Note: Notations and Terminologies are standard.

Q.1 Choose correct option from given four choices. [08]

(i) Let $x, y \in \mathbb{R}^n$ be orthogonal (i.e. perpendicular) vectors. Then $\|x+y\|^2 =$

(a) $\|x\|^2 + \|y\|^2$ (b) $\|x\| + \|y\|$ (c) $(\|x\| + \|y\|)^2$ (d) $\|x\|\|y\|$

(ii) Which of the following is true?

(a) $\lim_{x\to 0} x\cos(\frac{1}{x}) = 0$ (b) $\lim_{x\to 0} \frac{\sin x}{x} = 0$ (c) $\lim_{x\to 0} \frac{\cos x}{x} = 1$ (d) none

(iii) Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be defined as $f(t) = 3t^3$. Then $Df(2) =$

(a) λ_{12} (b) λ_{24} (c) λ_{36} (d) λ_{48}

(iv) Let $a = (2, 1)$ and $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ be defined as $f(x) = x_1x_2$. Then $Df(a) =$

(a) $\pi_1 + \pi_2$ (b) $2\pi_1 + \pi_2$ (c) $\pi_1 + 2\pi_2$ (d) none

(v) Define $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ as $f(x) = 2x_1e^{x_2}$. Then $D_1f(0) =$

(a) -1 (b) 0 (c) 1 (d) 2

(vi) Let $a \in \mathbb{R}^n$ be fixed. Let $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ be continuous at a. Then

(a) $Df(a)$ exists (b) $D_xf(a)$ exist (c) $D_jf(a)$ exist (d) none

(vii) Let $S \in \mathcal{T}^3(V)$ and $T \in \mathcal{T}^5(V)$. Then $S \otimes T$ belongs to

(a) $\mathcal{T}^{15}(V)$ (b) $\mathcal{T}^8(V)$ (c) $\mathcal{T}^5(V)$ (d) $\mathcal{T}^2(V)$

(viii) The dimension of $\Lambda^6(\mathbb{R}^4)$ is

(a) 0 (b) 15 (c) 24 (d) 4096

Q.2 Attempt any seven. [14]

(i) Prove that $\|x\| \le \sum_{i=1}^n |x_i|$ $(x \in \mathbb{R}^n)$.

(ii) Define oscillation $o(f; a)$ of $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ at a.

(iii) Define $f : \mathbb{R} \longrightarrow \mathbb{R}$ as $f(x) = x\cos(x)$. Find $Df(0) : \mathbb{R} \longrightarrow \mathbb{R}$.

(iv) Let $f : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ be differentiable at $a \in \mathbb{R}^n$. Then prove that its each component function $f^i : \mathbb{R}^n \longrightarrow \mathbb{R}$ is differentiable at a.

(v) If $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ has maximum value at a and $D_if(a)$ exists, then show that $D_if(a) = 0$.

(vi) Let $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ and $a, x \in \mathbb{R}^n$. Show that $D_{sx}f(a) = sD_xf(a)$ $(s \in \mathbb{R})$.

(vii) Define $T(x, y) = x_1y_2$ $(x, y \in \mathbb{R}^2)$. Find Alt(T).

(viii) Define tensor product and wedge product.

(ix) Let $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ be differentiable. Define $df(p)(v_p) := Df(p)(v)$ $(p \in \mathbb{R}^n;\ v_p \in \mathbb{R}^n_p)$. Prove that $df(p) : \mathbb{R}^n_p \longrightarrow \mathbb{R}$ is linear.

(Continue on page-2)

Q.3

(a) Let $x, y \in \mathbb{R}^n$. Then Prove that $|\langle x, y\rangle| \leq \|x\|\|y\|$ and $\|x+y\| \leq \| x\| + \|y\|$. [6]

(b) Let $A \subset \mathbb{R}^n$, let $f : A \longrightarrow \mathbb{R}$ be a bounded function, and let $a \in A$. Then prove that f is continuous at a iff $o(f; a) = 0$. [6]

OR

(b) Let $T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ be a linear map. Prove that there exists an $n \times m$ matrix A such that $T(x) = xA$ $(x \in \mathbb{R}^n)$. Further, if $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ is defined as $T(x) = (x_1+x_3, x_1-2x_2+x_3)$, then find A corresponding to T. [6]

Q.4

(a) If a function $f : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ is differentiable at $a \in \mathbb{R}^n$, then prove that there exists a unique linear transformation $\lambda : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ such that $\lim\limits_{h\to 0} \frac{\|f(a+h)-f(a)-\lambda(h)\|}{\|h\|} = 0$. [6]

(b) Let $f, g : \mathbb{R}^n \longrightarrow \mathbb{R}$ be differentiable at a. Prove that $f+g$ and fg are differentiable at a. [6]

OR

(b) Define $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ by

$$f(x) = \begin{cases} \frac{x_1|x_2|}{\|x\|} & (\text{if } x \neq 0) \\ 0 & (\text{if } x = 0) \end{cases}$$

Discuss the differentiability of f at 0. If it is differentiable at 0, then find its derivative. [6]

Q.5

(a) Let $a = (1, 0)$. Define $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ as $f(x) = (e^{x_1}, x_1 + \sin(x_2), \log(x_1) - x_2)$. Then find $f'(a)$ and $Df(a)$. [6]

(b) Prove that continuously differentiable function is differentiable. [6]

OR

(b) Let $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ be differentiable at a. Prove that $D_x f(a)$ exists for any $x \in \mathbb{R}^n$. Moreover, find $D_x f(0)$ for the function $f : \mathbb{R}^3 \longrightarrow \mathbb{R}$ defined as $f(y) = 2y_1 + y_2^2$. [6]

Q.6

(a) Define $Alt(T)$. Prove that if $T \in \mathcal{T}^k(V)$, then $Alt(T) \in \Lambda^k(V)$. [6]

(b) Let $\omega \in \Lambda^k(V)$ and $\eta \in \Lambda^l(V)$. Then prove that $\omega \wedge \eta = (-1)^{kl}(\eta \wedge \omega)$. [6]

OR

(b) Define "k-form" on $\mathbb{R}^n$. Let $f : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ be differentiable. Then prove that [6]

$$\tilde{f}_{1*}(d\pi_i) = \sum_{j=1}^{n} D_j f^i \cdot d\pi_j \ \ (1 \leq i \leq m).$$

THE END

Sc

(184) SEAT NO.______

No. of printed pages: 2

SARDAR PATEL UNIVERSITY

M.Sc. (Mathematics) Semester - I Examination
Thursday, 9^{th} November, 2017
PS01CMTH24, Linear Algebra

Time: 02:00 p.m. to 05:00 p.m. **Maximum marks: 70**

Note: 1. Figures to the right indicate full marks of the respective question.
2. Assume standard notations wherever applicable.

Q-1 Fill up the gaps in the following: [8]

1. The dimension of a vector space V is 9. Then $\dim \hat{V}$ =_____.
   (a) 3 (b) 9 (c) 27 (d) 81
2. Let V be any vector space over a field F and W be its subspace. Let $f \in \hat{V}$. Then _____ is not a subspace of V.
   (a) $\{0\}$ (b) $\ker f$ (c) $(\ker f) \cap W$ (d) W^0
3. Let V be a vector space over a field F. Then _____ is not an algebra over F.
   (a) $F_n[x]$ (b) $\text{Hom}(V,V)$ (c) $M_n(F)$ (d) F
4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(x_1,x_2) = (-x_2,-x_1)$, $(x_1,x_2) \in \mathbb{R}^2$. Then the characteristic roots of T are _____.
   (a) -1 and 1 (b) 0 and 2 (c) 0 and 1 (d) 1 and 1
5. If $T \in A(\mathbb{R}^5)$ is nilpotent with invariants $3,1,1$, then index of nilpotence of T is _____.
   (a) 5 (b) 3 (c) 2 (d) 0
6. Let V be a vector space over F and $T \in A(V)$ be nilpotent. Then $T^3 + 5T^2 + 3T$ is _____.
   (a) regular (b) onto (c) one-one (d) nilpotent
7. Let $A \in M_n(F)$ be regular and $\alpha \in F$, $\alpha \neq 0$. Then $\det(\alpha A^{-1})$ =_____.
   (a) $\frac{\alpha^n}{\det(A)}$ (b) $\frac{\alpha}{\det(A)}$ (c) $\frac{\det(A)}{\alpha^n}$ (d) $\alpha^n \det(A)$
8. Let F be a field and $A \in M_n(F)$ be nilpotent. Then $\text{tr}(A^7)$ =_____.
   (a) 0 (b) 1 (c) 7 (d) cannot be determined

Q-2 Attempt *Any Seven* of the following: [14]

(a) Let U, W be subspaces of a vector space V. If $U \subset W$ then show that $W^0 \subset U^0$.

(b) Let $W = \{(x_1,x_2,x_3) \in \mathbb{R}^3 : x_1 + x_2 + x_3 = 0\}$. Show that W is a subspace of $\mathbb{R}^3$ over $\mathbb{R}$.

(c) Let V be a finite dimensional vector space over F and $T \in A(V)$. Show that $\lambda \in F$ is a characteristic root of T if and only if $T - \lambda I$ is singular.

(d) Define $T : \mathbb{R}^3 \to \mathbb{R}^3$ by $T(x_1,x_2,x_3) = (x_1 - x_2 - x_3, x_2 - x_1, x_3)$, $(x_1,x_2,x_3) \in \mathbb{R}^3$. Find the matrix of T with respect to standard basis of $\mathbb{R}^3$.

(e) Let V be a vector space over a field F and $T \in A(V)$. Show that $\ker(T)$ is invariant under T.

(f) Let V be a vector space over F and $S, T \in A(V)$ be nilpotent. Show that $S + T$ is nilpotent.

(g) Let $A \in M_n(F)$ be invertible. Show that $\det(A) \neq 0$.

(h) For $A, B \in M_n(\mathbb{R})$, show that $\text{tr}(AB) = \text{tr}(BA)$.

(i) Find the symmetric matrix associated with the following quadratic form: $9x^2 - y^2 + 4z^2 + 6xy - 8xz + 2yz$.

Q-3 (a) Let V be a finite-dimensional vector space over a field F. If W is a subspace of V then show that W is also finite-dimensional and $\dim V/W = \dim V - \dim W$. [6]

(b) Let V and W be vector spaces over F of dimensions m and n respectively. Prove that $\dim \text{Hom}(V, W) = mn$ over F. [6]

OR

(b) Let V be a finite dimensional vector space over F. Show that V is isomorphic to $\hat{\hat{V}}$. [6]

Q-4 (a) Let V be a vector space over F and $T \in A(V)$. Show that characteristic vectors corresponding to distinct characteristic roots of T are linearly independent. [6]

(b) Let V be a vector space over F and $T \in A(V)$. Show that T is regular if and only if the constant term of the minimal polynomial for T is non-zero. [6]

OR

(b) Let V be a vector space over F, $T \in A(V)$, and $B_1 = \{v_1, v_2, \dots, v_n\}$ and $B_2 = \{w_1, w_2, \dots, w_n\}$ be bases of V. If $m_1(T)$ and $m_2(T)$ are matrices of T with respect to the bases B_1 and B_2 respectively, then show that $m_1(T)$ and $m_2(T)$ are similar. [6]

Q-5 (a) Let V be a finite dimensional vector space over F and $T \in A(V)$. Let $p(x) = (q_1(x))^{l_1}(q_2(x))^{l_2} \cdots (q_k(x))^{l_k}$ be the minimal polynomial for T, where $q_i(x) \in F[x]$ is irreducible, $i = 1, \dots, k$. Let $V_i = \ker(q_i(T))^{l_i}$. Show that each $V_i \neq \{0\}$. Assuming that V_i is invariant under T, show that $V = V_1 \oplus V_2 \oplus \cdots \oplus V_k$. [6]

(b) Let V be a finite dimensional vector space over F, $T \in A(V)$ be nilpotent with index of nilpotence k. Let $v \in V$ such that $T^{k-1}v \neq 0$. Show that $W = L(\{v, Tv, \dots, T^{k-1}v\})$ is invariant under T and $\dim W = k$. [6]

OR

(b) Let V be a finite dimensional vector space over F and $T \in A(V)$ be nilpotent. Then show that the invariants of T are unique. [6]

Q-6 (a) For $A, B \in M_n(F)$, show that $\det(AB) = \det(A)\det(B)$. [6]

(b) i. State and prove Jacobson lemma. [4]

ii. Find the inertia $In(A) = (p, q, k)$ of the symmetric matrix A associated with the quadratic form: $11{x_1}^2 + 6x_1x_2 + 19{x_2}^2 = 80$. [2]

OR

(b) Let F be a field of characteristic 0, V be a vector space over F and $T \in A(V)$. If $\text{tr}(T^i) = 0$ for all $i \geq 1$ then show that T is nilpotent. [6]

################

— x —

(2)

SC

(173)

No of printed pages: 2

Sardar Patel University

M.Sc. (Sem-I), PS01CMTH25, Methods of Differential Equations;
Saturday, 11^{th} November, 2017; 02.00 p.m. to 05.00 p.m.

Maximum Marks: 70

Note: (i) Notations and terminologies are standard; (ii) Figures to the right indicate marks.

Q.1 Answer the following. [8]

1. The degree of differential equation $y'' - \sqrt{y'} = 0$ is
   (A) 1 (B) $\frac{1}{2}$ (C) 2 (D) 4
2. The set of ordinary points of $(1-x)y'' + (x-1)y' + (1-e^{x-1})y = 0$ is
   (A) $\{0\}$ (B) φ (C) $\{1\}$ (D) none of these
3. $\int_{-1}^{1} J_3(x)dx =$
   (A) $\sqrt{\pi}$ (B) 0 (C) -1 (D) none of these
4. $\int_{-1}^{1} x^2 P_2(x)dx =$
   (A) 0 (B) $\frac{2}{15}$ (C) $\frac{4}{15}$ (D) none of these
5. Which of the following is not an integrating factor of $ydx - xdy$?
   (A) $\frac{1}{y^2}$ (B) $\frac{1}{xy}$ (C) $\frac{1}{x^2}$ (D) none of these
6. Which one is not homogeneous Pfaffian differential equation?
   (A) $xdx + ydy + zdz = 0$
   (B) $(x^2+1)dx + (y^2+1)dy + (z^2+1)dz = 0$
   (C) $xydx + yzdy + zxdz = 0$
   (D) none of these
7. $F(-1, \frac{1}{3}; \frac{1}{3}; -1) =$
   (A) 1 (B) 3 (C) -1 (D) none of these
8. The radius of convergence of Gauss's hypergeometric series is
   (A) 0 (B) 1 (C) 2 (D) $\frac{1}{2}$

Q.2 Attempt any ***seven***: [14]

(a) Find the interval of convergence of $\sum_{n=0}^{\infty} \frac{n}{n+1} x^n$.
(b) State Frobenius theorem.
(c) Define gamma function and find value at 1.
(d) Find $J_{-\frac{1}{2}}(x)$.
(e) Using Rodrgue's formula find P_0 and P_1.
(f) State Picard's theorem.
(g) Find $F(1,1;2;x)$.
(h) Find a partial differential equation by eliminating F from $z = F(\frac{xy}{z})$.
(i) State the necessary and sufficient condition that Pfaffian differential equation in three variables is integrable.

(1)

(P.T.O.)

Q.3

(a) Solve: $x^2y'' - xy' - (x-1)y = 0$ near 0. [6]

(b) Classify singularities: $x(x-1)^2(x+2)y'' + x^2(x-1)y' + (x+2)y = 0$ [6]

OR

(b) Solve: $y'' + (x-1)y' + y = 0$ near 1.

Q.4

(a) State and prove orthogonality of Legendre's polynomials. [6]

(b) Prove: $2\alpha J_\alpha(x) = x[J_{\alpha-1}(x) + J_{\alpha+1}(x)]$. [6]

OR

(b) Show that $x^2 = \sum_{n=1}^{\infty} \frac{2}{\lambda_n J_3(\lambda_n)} J_2(\lambda_n x)$, $x \in (0,1)$, where $\{\lambda_n\}$ is a sequence of positive roots of $J_2(x)$.

Q.5

(a) Solve $y' - 2(x+xy) = 0$, $y(0) = 1$ using Picard's method of successive approximations. [6]

(b) State and prove integral representation of Gauss's hypergeometric function. [6]

OR

(b) Show that $F\left(\alpha, \beta; \beta-\alpha+1; -1\right) = \dfrac{\Gamma(1+\frac{\beta}{2})\,\Gamma(1+\beta-\alpha)}{\Gamma(1+\beta)\,\Gamma(\frac{\beta}{2}-\alpha+1)}$.

Q.6

(a) Show that $X\cdot\text{curl}X = 0$ iff $\mu X\cdot\text{curl}(\mu X) = 0$ where $X = (P, Q, R)$ and $P, Q, R, \mu(\neq 0)$ are functions of x, y and z. [6]

(b) Solve: $(z^2 - 2yz - y^2)p + x(y+z)q = x(y-z)$. [6]

OR

(b) Verify that the differential equation $(y^2+z^2)dx + xydy + xzdz = 0$ is integrable and find its primitive.

* * * * *

8c

(97) SEAT No.________

No. of printed pages: 2

SARDAR PATEL UNIVERSITY
M. Sc. (Semester I) Examination

Date: 14-11-2017, Tuesday **Time: 2.00 To 5.00 p.m.**
Subject: MATHEMATICS **Paper No. PS01EMTH01 – (Graph Theory – I)**

Total Marks: 70

1. Choose the correct option for each question: [8]

(1) For $G = K_{n,m}$, $(m, n \geq 2)$ if diam(G) = d and rad(G) = r, then
(a) d = r (b) d < r (c) d > r (d) none of these

(2) For $G = C_n$ with anticlockwise direction, rank(B) is
(a) n (b) n – 1 (c) 1 (d) none of these

(3) Let T be a spanning out-tree with root R. Then
(a) $d^+(R) > 0, d^-(R) > 0$ (c) $d^+(R) = 0, d^-(R) = 0$
(b) $d^-(R) = 0, d^+(R) > 0$ (d) $d^+(R) = 0, d^-(R) > 0$

(4) If G is a simple digraph with vertices $\{v_1, v_2, \ldots, v_n\}$ & e edges, then $\sum_{i=1}^{n} d^-(v_i) =$
(a) ne (b) 2e (c) e (d) e^2

(5) The chromatic number of C_{2m} $(m \in \mathbb{N})$ is
(a) 2 (b) 3 (c) m (d) 2m

(6) Which of the following graphs is Hamiltonian?
(a) $K_{n,1}$ (b) $K_{n,2n}$ (c) C_n (d) P_n

(7) Let G be a simple graph without isolated vertex. Then a matching M in G is
(a) maximum ⇒ perfect (c) maximal ⇒ maximum
(b) maximal ⇒ perfect (d) maximum ⇒ maximal

(8) For which of the following graphs, $\alpha(G) = \beta(G)$?
(a) K_5 (b) C_6 (c) P_5 (d) none of these

2. Attempt any SEVEN: [14]

(a) Prove: If $K_{m,n} = K_{m+n}$, then m = n = 1.
(b) Prove or disprove: A regular digraph is strongly connected.
(c) Define adjacency matrix in a digraph.
(d) Give an example of a spanning in tree which is also a spanning out tree in a digraph.
(e) Is K_5 uniquely colorable? Why?
(f) What is Four color problem?
(g) Prove or disprove: The graph P_4 is isomorphic to $K_{1,3}$.
(h) Prove or disprove: The graph $K_{2,3}$ has a perfect matching.
(i) Prove or disprove: Every independent set is a vertex cover of the graph.

(P.T.O.)

3. (a) Define the following with examples: [6]
(i) Symmetric digraph (ii) complete symmetric digraph (iii) Asymmetric digraph (iv) complete asymmetric digraph

(b) Prove: If G is a connected Euler digraph, then it is balanced. [6]

OR

(b) Obtain De Bruijn cycle for $r = 3$. [6]

4. (a) Define arborescence and show that an arborescence is a tree in which every vertex other than the root has an in-degree exactly one. [6]

(b) Show that the determinant of every square sub matrix of the incidence matrix A of a digraph is 1, −1 or 0. [6]

OR

(b) Let G be a connected digraph with n vertices. Prove that rank of $A(G) = n - 1$. [6]

5. (a) Prove: If G is Hamiltonian, then, for each $S \subset V(G)$, $c(G - S) \leq |S|$. [6]

(b) Prove: A connected graph G is 2-chromatic if and only if it does not contain an odd cycle. [6]

OR

(b) Find the Chromatic polynomial of the graph $K_{1,3}$. [6]

6. (a) Prove: If G is a bipartite graph, then $\alpha'(G) = \beta(G)$. [6]

(b) Define a matching in a graph and show that every component of a symmetric difference of two matching is either a path or a cycle of even length. [6]

OR

(b) Define $\alpha(G)$, $\beta(G)$ and find it with the corresponding sets for $G = P_7$. [6]

x-x-x-x-x-x

(2)

SC

[98] SEAT No.________ NO. OF PRINTED PAGES: 2

SARDAR PATEL UNIVERSITY
M.Sc. (Semester-I) Examination

Tuesday 14/11/2017 **Time: 02:00 PM to 05:00 PM**
Subject: Mathematics **Course No.PS01EMTH02**

Mathematical Classical Mechanics

Note:
(1) All questions (including multiple choice questions) are to be answered in the answer book only.
(2) Numbers to the right indicate full marks of the respective question.

Q-1 Choose most appropriate answer from the options given. (08)

(1) A particle is moving on a cylinder, its degrees of freedom is________ .
(a) 0 (b) 2 (c) 4 (d) can not be determined

(2) The motion of a particle under gravity is ___________ constraint.
(a) not a (b) a holonomic (c) a non-holonomic (d) conservative

(3) The condition for extremum of $J = \int_{x_1}^{x_2} f(y,x)dx$ is _______ .
(a) $\frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}}\right) = 0$ (b) $\frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}}\right) - \frac{\partial f}{\partial y} = 0$
(c) $\frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}}\right) - \frac{\partial f}{\partial \dot{y}} = 0$ (d) none of these

(4) If the Lagrangian L does not depend on q_j explicitly then ___________ is conserved.
(a) p_j (b) h (c) $\dot{p}_j$ (d) L

(5) Which one of the following is correct?
(a) $\frac{\partial L}{\partial t} = \frac{dh}{dt}$ (b) $H = h$ (c) $\frac{dL}{dt} = \frac{dH}{dt}$ (d) none of these

(6) If all coordinates are non-cyclic then Routhian $R =$ ________.
(a) H (b) L (c) $-H$ (d) 0

(7) Pick up the incorrect statement.
(a) A canonical transformation is non-invertible.
(b) Jacobian matrix for a canonical transformation is symplectic.
(c) Inverse of a canonical transformation is canonical.
(d) None of the above.

(8) $[q_1, p_2]$ is __________.
(a) a fundamental Lagrange bracket (b) a fundamental Poisson bracket
(c) a zero matrix (d) an undefined term

Q-2 Answer any **Seven.** (14)

(1) Define and give an example of a rheonomic constraint.
(2) Describe constraints in Atwood's machine.
(3) What are geodesics on a unit sphere?
(4) Define generalized momentum conjugate to a generalized coordinate.
(5) Explain the meaning of Legendre transformation in brief.
(6) State principle of least action.
(7) State the transformation generated by a function of type F_2.
(8) Define Poisson bracket.
(9) Evaluate $\{p_1, q_1 + p_2\}$, notations being usual.

(P.T.O)

Q-3

(a) State Lagrange's equations of motion in general for and derive the form in the case of velocity dependent potential. (06)

(b) Giving all details obtain expression of Lagrangian for spherical pendulum. (06)

OR

(b) Express kinetic energy of system in terms of generalized coordinates and velocities.

Q-4

(a) Derive the condition for the extremum of $J = \int_{x_1}^{x_2} f(y, \dot{y}, x)dx$. (06)

(b) Using calculus of variations discuss brachistochrone problem. (06)

OR

(b) Lagrangian of a system is given by $L = \frac{1}{2}(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{1}{r^2}$. Compute all generalized momenta and energy function. Which of them are conserved? Why?

Q-5

(a) State Hamilton's modified principle; and derive Hamilton's equations of motion from it. (06)

(b) Giving an example describe Routhian procedure. (06)

OR

(b) Find Hamiltonian corresponding to the Lagrangian,
$L = a\,\dot{x}^2 + b\frac{\dot{y}}{x} + c\dot{x}\,\dot{y} + f\,y^2\dot{x}\,\dot{z} + g\,\dot{y} - k\sqrt{x^2 + y^2}$,
where a, b, c, f, g and $k$are constants; x, y and z are generalized coordinates.

Q-6

(a) Define fundamental Lagrange brackets. Show that they are invariant under a canonical transformation. (06)

(b) Define infinitesimal canonical transformation. Show that symplectic condition is satisfied in this case. (06)

OR

(b) Show that the transformation,
$Q = \log(1 + \sqrt{q}\cos p)\,, P = 2\sqrt{q}\,(1 + \sqrt{q}\cos p)\sin p$,
is canonical.

****

SC

(99)

SEAT No.________

No. of printed pages: 2

SARDAR PATEL UNIVERSITY

M. Sc. (Semester I) Examination

Date: 14-11-2017, Tuesday **Time: 2.00 To 5.00 p.m.**

Subject: MATHEMATICS **Paper No. PS01EMTH21 – (Graph Theory – I)**

Total Marks: 70

1. Choose the correct option for each question: [8]

(1) For $G = K_{n,m}$, $(m, n \geq 2)$ if diam(G) = d and rad(G) = r, then
(a) d = r (b) d < r (c) d > r (d) none of these

(2) The order of an incidence matrix of a digraph P_n is
(a) n x n (b) (n – 1) x n (c) n x (n – 1) (d) (n –1) x (n–1)

(3) Let T be a spanning in-tree with root R. Then
(a) $d^+(R) > 0, d^-(R) > 0$ (c) $d^+(R) = 0, d^-(R) = 0$
(b) $d^-(R) = 0, d^+(R) > 0$ (d) $d^+(R) = 0, d^-(R) > 0$

(4) If G is a simple digraph with vertices $\{v_1, v_2, \ldots, v_n\}$ & e edges, then $\sum_{i=1}^{n} d^-(v_i) =$
(a) ne (b) e (c) 2e (d) e^2

(5) The chromatic number of C_{2m} $(m \in \mathbb{N})$ is
(a) 2 (b) 3 (c) m (d) 2m

(6) Which of the following graphs is not Hamiltonian?
(a) K_n (b) $K_{n, n}$ (c) P_n (d) C_n

(7) Let G be a simple graph without isolated vertex. Then a matching M in G is
(a) maximum ⇒ perfect (c) maximal ⇒ maximum
(b) maximal ⇒ perfect (d) maximum ⇒ maximal

(8) For which of the following graphs, $\alpha(G) = \beta(G)$?
(a) K_4 (b) C_6 (c) P_5 (d) none of these

2. Attempt any SEVEN: [14]

(a) Prove: If $K_{m, n} = K_{m+n}$, then m = n = 1.
(b) Prove or disprove: A regular digraph is strongly connected.
(c) Define adjacency matrix in a digraph.
(d) Prove or disprove: Every connected digraph has a spanning out-tree.
(e) Is K_6 uniquely colorable? Why?
(f) What is Four color problem?
(g) Define isomorphic graphs and give one example of it.
(h) Prove: If $S \subset V(G)$ is an independent set, then V(G) – S is vertex cover of G.
(i) Prove or disprove: If K_n has a perfect matching, then n is even.

(P.T.O.)

3. (a) Define the following with examples:
(i) In – degree and out- degree (ii) Symmetric and asymmetric digraph [6]

(b) Prove: If G is a connected Euler digraph, then it is balanced. [6]

OR

(b) Prove: If G is a simple graph, then G has at least two vertices u & v with $d(u) = d(v)$. [6]

4. (a) Define arborescence and show that an arborescence is a tree in which every vertex other than the root has an in-degree exactly one. [6]

(b) Show that the determinant of every square sub matrix of the incidence matrix A of a digraph is 1, −1 or 0. [6]

OR

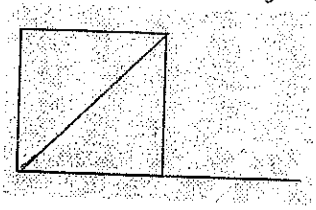
(b) Define fundamental circuit matrix in a digraph G and if B denotes the circuit matrix of G with n vertices and e edges, then prove that $\text{rank}(B) \geq e - n + 1$. [6]

5. (a) Prove: If G is simple graph with $n = |V(G)| \geq 3$ & $2\delta(G) \geq n$, then G is Hamiltonian. [6]

(b) Prove: A connected graph G is 2-chromatic if and only if it does not contain an odd cycle. [6]

OR

(b) Find the coefficients c_3 and c_4 of Chromatic polynomial of the graph below: [6]



6. (a) Prove: If G is a bipartite graph, then $\alpha'(G) = \beta(G)$. [6]

(b) State Hall's theorem and show that a k-regular bipartite graph has a perfect matching. [6]

OR

(b) Define $\alpha(G)$, $\beta(G)$ and find it with the corresponding sets for $G = C_7$. [6]

x-x-x-x-x-x

SC

SEAT No.________

[100] NO. OF PRINTED PAGES: 2

SARDAR PATEL UNIVERSITY
M.Sc. (Semester-I) Examination

Tuesday 14/11/2017 Time: 02:00 PM to 05:00 PM

Subject: Mathematics Course No.PS01EMTH22

Mathematical Classical Mechanics

Note:
(1) All questions (including multiple choice questions) are to be answered in the answer book only.
(2) Numbers to the right indicate full marks of the respective question.

Q-1 Choose most appropriate answer from the options given. (08)

(1) A particle is moving on a sphere the degrees of freedom is________ .
(a) 0 (b) 1 (c) 2 (d) can not be determined

(2) Motion of two particles connected by a rigid rod is __________ constraint.
(a) not a (b) a holonomic (c) a rheonomic (d) conservative

(3) The condition for extremum of $J = \int_{x_1}^{x_2} f(\dot{y}, x)dx$ is _______ .
(a) $\frac{\partial f}{\partial \dot{y}} = constant$ (b) $\frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}}\right) = \frac{\partial f}{\partial y}$
(c) $\frac{d}{dy}\left(\frac{\partial f}{\partial \dot{y}}\right) - \frac{\partial f}{\partial \dot{y}} = constant$ (d) none of these

(4) If the Lagrangian L does not depend on q_j explicitly then __________ is conserved.
(a) p_j (b) h (c) $\dot{p}_j$ (d) L

(5) Which one of the following is not correct?
(a) $\frac{\partial H}{\partial t} = \frac{dh}{dt}$ (b) $H = h$ (c) $\frac{dL}{dt} = \frac{dH}{dt}$ (d) none of these

(6) If all coordinates are cyclic then Routhian $R =$ _______.
(a) H (b) L (c) $-H$ (d) 0

(7) Pick up the correct statement.
(a) A canonical transformation is non-invertible.
(b) Jacobian matrix for a canonical transformation is symplectic.
(c) Inverse of a canonical transformation is canonical.
(d) None of the above.

(8) $\{q_1, q_2\}$ is _________.
(a) a fundamental Lagrange bracket (b) a fundamental Poisson bracket
(c) a zero matrix (d) an undefined term

Q-2 Answer any **Seven**. (14)

(1) Define and give an example of a scleronomic constraint.
(2) Compute degrees of freedom for a simple pendulum.
(3) State the condition for extremum of
$J = \int_{x_1}^{x_2} f(y_1, y_2, \dots, y_n, \dot{y}_1, \dot{y}_2, \dots \dot{y}_n, x)dx$
(4) Define a cyclic coordinate.
(5) Explain the meaning of Legendre transformation in brief.
(6) State Hamilton's equations of motion in matrix form.
(7) State the transformation generated by a function of type F_1.
(8) State fundamental Poisson brackets.
(9) Prove linearity of Lagrange brackets.

(P.T.O.)

Q-3

(a) State D'Alembert's principle and derive Lagrange's equations of motion in general form. (06)

(b) Giving all details obtain Lagrange's equations of motion for Atwood's machine. (06)

OR

(b) A particle is moving inside the unit circle. Express the constraints in this case in mathematical form and hence classify them.

Q-4

(a) State Hamilton's principle and hence derive Lagrange's equations from it. (06)

(b) Using calculus of variations obtain geodesics on plane. (06)

OR

(b) Prove the law of conservation of linear momentum using Lagrangian formalism.

Q-5

(a) Derive Lagrange's equations of motion from Hamilton's equations of motion. (06)

(b) Derive Routhian equations for the system having Lagrangian $L = \frac{1}{2}(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{1}{r^2}$. (06)

OR

(b) Find Hamiltonian corresponding to the Lagrangian,
$L = \frac{I_1}{2}(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta) + \frac{I_3}{2}(\dot{\psi} + \dot{\phi}\cos\theta)^2 - Mgl\cos\theta$,
notations being usual.

Q-6

(a) State and prove Jacobi's identity for Poisson brackets. (06)

(b) Obtain symplectic condition for canonical transformation. (06)

OR

(b) Show that the transformation, $e^Q = \frac{\sin p}{q}, P = q\cot p$, is canonical.

****

— x —