

[197/198/A-30]

SEAT No. _____

No. of printed pages: 04

SARDAR PATEL UNIVERSITY
M.Sc(Statistics) Fourth Semester and Certificate
Course in Financial Statistics Examinations
Thursday, March 28, 2019
2:00 p.m to 5:00 p.m

Subject: PS04ESTA02/ PS04ESTA22: Actuarial Statistics

Note: Figure to the right indicate full marks to the questions. Total Marks 70

1. Choose the correct answer(s) and write in your answer book. 8
- (i) The probability that the house will not be damaged by fire in a period of one year is p and the probability density function of positive loss is exponential with mean θ units. Then expected loss is
- (a) θ units (b) $p(1-p)\theta$ units
(c) $p\theta$ units (d) $(1-p)\theta$ units
- (ii) Utility function is a function which measures
- (a) The value of utility attached to wealth (b) Utility of property
(c) Utility of time (d) None of (a) to (c)
- (iii) Which of the following is not a survival function for $x \geq 0$
- (a) $S(x) = \frac{x}{\theta}, 0 < x < \theta.$ (b) $S(x) = \exp(-x^2), x > 0.$
(c) $S(x) = \frac{1}{(1+x)^2}, x > 0.$ (d) $S(x) = \exp(-x), x > 0.$
- (iv) If $S_N = X_1 + X_2 + \dots + X_N$ is random sum of i.i.d random variables, distributed like X , which are independent of N then variance of S_N is
- (a) $\text{Var}(X)E(N)$ (b) $E(X^2)\text{Var}(N)$
(c) $\text{Var}(X)E(N) + E(X^2)\text{Var}(N)$ (d) None of these
- (v) $t|u|q_x$ stands for
- (a) Probability that (x) will die between ages $x+t$ and $x+t+u$ (b) $P[t < T(x) \leq t+u]$
(c) $P[x+t < X \leq x+t+u | X > x]$ (d) all (a) to (c)
- (vi) If curtate random Variable $K(x)=8$ then residual life random variable $T(x)$ takes values:
- (a) $8 < T(x) \leq 9$ (b) $7 \leq T(x) \leq 8$
(c) $8 \leq T(x) < 9$ (d) None of these
- (vii) Suppose life length random variable is modeled by uniform distribution over the

(1)

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interval $(0, \omega)$. Which following relations is correct?

(a) $d_x = l_x - l_{x+1}$

(b) $d_x = \frac{l_0}{\omega}$

(c) both (a) and (b)

(d) None of (a) to (c)

(viii) In annuities

(a) payments occur at the end of term

(b) payments occur in regular intervals

(c) both (a) and (b)

(d) none of (a) to (c)

2 Answer any seven of the following.

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- (i) Explain with examples role of risks in insurance.
 (ii) What do you mean by relative security loading?
 (iii) What do you understand by utility function? Give desirable properties for a utility function.
 (iv) Suppose a survival model is defined by the following values of p_x .

x	0	1	2	3	4
p_x	0.9	0.8	0.6	0.3	0

What are corresponding values of $S(x)$?

(v) Under the Bulducci assumption show that

$$P(T(x) > t) = \frac{1 - q_x}{1 - (1 - t)q_x}$$

- (vi) In the usual notation obtain the expression for ${}_{1.5}P_{30.5}$ in terms of l_x .
 (vii) Show that $L_x = \int_x^{x+1} l_t dt$
 (viii) If $l_{x+t} = a/(3+t)$, $0 \leq t \leq 1$, $l_{x+1/2} = 800$, evaluate $l_{x+3/4}$, without finding a.
 (ix) Obtain the expression for $S_{\overline{n}|}$.

(2)

- 3 (a) Suppose survival model is defined for certain kind of insects, expected to live maximum six weeks, by the following values of p_x . 6
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|---------|-----|-----|-----|-----|-----|-----|---|
| x: | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| p_x : | 0.9 | 0.8 | 0.7 | 0.5 | 0.3 | 0.1 | 0 |

Using radix $l_0 = 10000$ construct life table.

- (b) From the life table constructed in (a) answer the following 6
- (i) If l_0 insects born every week, what is the total population?
 - (ii) What is proportion of insects aged 3 weeks, expected live 5 weeks?
 - (iii) What is the expected number of insects living between ages 1 and 4 weeks?

OR

- (b) Write a detailed note on insurance business in India 6
- 4 (a) Describe De Moivre' law, Gompertz Law and Makeham's Law and Weibull law by giving their force of mortality and obtaining survival functions. 6
- (b) Describe n-Year Pure Endowment Insurance and obtain the expression for mean and variance of the present value random variable Z in actuarial notation. 6

OR

- (b) If $l_x = 1000\sqrt{(100 - x)}$, $0 \leq x \leq 100$, calculate exact value of $\mu_{36+\frac{1}{4}}$. Also find it under the assumption of uniformity of deaths in a unit interval. Compare the two values. 6
- 5 (a) Describe n year term life insurance scheme with the help of international notations for actuaries. Obtain expressions for net single premium and variance of Z_T , the present value at policy issue, of the benefit payment. 6
- (b) The probability density function of the future lifetime T for (x) is assumed to 6

$$\text{be } g(t) = \begin{cases} \frac{1}{80} & \text{if } 0 < t < 80 \\ 0 & \text{otherwise.} \end{cases},$$

At a force of interest δ , calculate for Z, the present-value random variable for a whole life insurance of unit amount issued to (x), (i) the net single (ii) the variance (c) the 90th percentile.

OR

- (b) Consider a 5-year deferred whole life insurance payable at the moment of death of (x). The individual is subject to a constant force of mortality $\mu = 0.04$. For the present value random variable X of the benefit payment at $\delta = 0.10$ (i) calculate the expectation and the variance, (ii) find the distribution function and (iii) calculate the median. 6
- 6 (a) Describe Annuities Certain Immediate. Find the present value and 6

accumulated value of 10 year annuity immediate of ₹ 1000 per annum if the effective rate of interest is 5% for the first two years , 6% for the next three years and 7% for rest five years.

- (b) Explain in detail mthly whole life annuity. 6

OR

- (b) Explain the concept of reserves as used in the insurance business. For n-year endowment insurance issued to (d) on a fully continuous basis, write down the expression for prospective loss random variable. Find its expectation and variance. Obtain the formula for reserve in this setup. 6

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