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SEAT No.________

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SARDAR PATEL UNIVERSITY
M.Sc. (II Semester) Examination
2018
Tuesday, 23rd October
10.00 am to 1:00 pm
STATISTICS COURSE No. PS02CSTA21/PS02CSTA01
(Stochastic Processes)

Note: Figures to the right indicate full marks of the questions. **(Total Marks: 70)**

1 Attempt all, write correct answers **08**

(i) For a finite, absorbing Markov chain with one absorbing state, the other state are
a) ergodic b) recurrent
c) transient d) none of these

(ii) $\{N(t), t\geq 0\}$ is a Poisson process with intensity rate 2/3, What is the P(N(0))=?
a) 2/3 b) 0
c) exp(-2/3) d) 1

(iii) A radioactive element emits with rate λ, and emission is recorded with probability 0.6. Then the rate of recorded emission is
a) 0.4λ b) 0.6λ
c) λ d) $0.4(1-\lambda)$

(iv) For a doubly stochastic Markov chain the unfilled elements in $\begin{bmatrix} .5 & .5 & 0 \\ 0 & .4 & \\ .5 & .1 & \end{bmatrix}$ are
a) 1, 0 b) .6, .4
c) .4, .6 d) None of these

(v) In usual notation, the $p_0(t)$ for any discrete state space Markov process is given by
a) P(0, s) b) P(s, 0)
c) P(0,t) d) P(s, t)

(vi) The ultimate extinction of a geometric branching process $\{X_n, n\geq 0\}$ having $X_0 = 1$ is sure when ____
a) $q<1$ b) $p\leq q$
c) $q\leq p$ d) $q<1/2$

(vii) If Weiner process has mean 0 and variance $\sigma^2 t$ then covariance between W(10) and W(5) is
a) $15\sigma^2$ b) $5\sigma^2$
c) $10\sigma^2$ d) None of these

(viii) The Ornstein-Uhlenbeck process is analogues of which of the following?
a) simple random walk process b) AR(2) process
c) AR(1) process d) Weiner process

2 Attempt ANY 7, each carries 2 marks **14**

(a) Give classification of stochastic processes, giving theoretical as well as a practical example.

(b) Derive Chapman-Kolmogorov equation. State its uses.

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(c) Write brief note on Markov chain having TPM $\begin{bmatrix} 1 & 0 & 0 \\ .3 & .4 & .3 \\ .2 & .1 & .7 \end{bmatrix}$.

(d) Define Ehrenfest Markov chain having 4 states.

(e) What is compound Poisson process and homogeneous Poisson process?

(f) Show that P{N(s)=k | N(t) = n] is related the binomial probability function where N(t) denotes Poisson process events in time interval t>0.

(g) State two important results of discrete branching process.

(h) Define pure birth process and obtain the first term in its probability distribution.

(i) Write down the partial –differential equations for the linear growth process having increment and death rates as, $\lambda_n = n\lambda$, $\mu_n = n\mu$.

(j) Define standard Weiner process. Also mention about its use.

3(a) Define Random Walk Model. Discuss its properties. 06

3(b) Show in usual notation for absorbing Markov chain that A = NR. 06

OR

3(b) State and prove a result about Gambler's ruin chain.

4(a) Define Poisson process. Show that sum of two Poisson processes is a Poisson process. Also derive the probability distribution of waiting times. 06

4(b) Let $X(t) = \sum_{i=1}^{N(t)} Y_i$ where N(t) is a Poisson process and Y's are iid random variables. Obtain mean and variance of X(t). 06

OR

4(b) Explain nonhomogeneous Poisson process. Why it is called a generalization of poisson process?

5(a) Explain birth and death process. Obtain its difference differential equations. 06

5(b) Define pure death process. Assuming suitable initial value show that its probability model is binomial with probability parameter $exp(-\mu t)$, $t \geq 0$, μ is death rate. 06

OR

5(b) What is Yule-Fury process? Obtain its probability generating function.

6(a) Define Weiner process and derive its transition probability density function. 06

6(b) Give definition of diffusion process along with that of infinitesimal conditional mean, variance. Also express the Kolmogorov's forward differential equation. 06

OR

6(b) Discuss suitability of each, the Weiner process and the Ornstein-Uhlenbeck process. State their important parameters' estimates.

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