[28] **SEAT No.**_____ *No. of Pages to be Printed:* 04

SARDAR PATEL UNIVERSITY

External Examination

M. Sc. Applied Statistics, Semester – II

March 23rd, 2019 (Saturday)

PS02CAST21: *Parametric Inference and Nonparametric Inference*

Time: **10:00 A.M. – 01:00 P.M.** *Marks:* 70

Q. 1. Multiple Choice Questions [Single-Correct-Answer type] (Attempt all) [8]

i. A test is said to be Unbiased if

a. Power of the test is maximum among the class of all possible tests

b. Size of the test is minimum among the class of all possible tests

c. Power of the test is at most equal to its size

d. Size of the test is at most equal to its power.

ii. Which of these is a necessary and sufficient condition for an Unbiased Estimator to be UMVUE?

a. The estimator in question must be uncorrelated with every other unbiased estimator for the parameter across the entire parameter space.

b. The estimator in question must have the highest relative efficiency with respect to any other unbiased estimator under most of the parameter space.

c. The estimator in question must be uncorrelated with any other biased estimator for the parameter across the entire parameter space.

d. The estimator in question must have the highest relative efficiency with respect to any other estimator across the entire parameter space.

iii. Consider the two statements about estimators of the mean of a Poisson(θ) population:

I. Moment Estimator = MLE

II. MVUE = UMVUE

III. MLE = MVBE

a. Only **III** is incorrect

b. Only **II** and **III** are incorrect

c. Only **I** and **III** are incorrect

d. None of the above

iv. A robber goes into a bank to rob money by using a gun to scare the employee at the counter. When he gets out of the bank with the heist money he realizes that he had no bullets in the gun in the first place. Given that the null hypothesis was "*There is at least one bullet in the gun*", what kind of error occurred inside the bank?

a. Type-I error

b. Type-II error

c. **a** from the point of view of the Banker and **b** from the point of view of the Robber.

d. Neither of the above

[Turn Over]

(P.T.O.)

v. When data types are Artificially Dichotomous we use the following Non-Parametric correlation test?

a. Point Bi-Serial
b. Tetrachoric
c. Phi coefficient
d. None of the above

vi. Range of Covariance is

a. (-1, 1)
b. $[0, \infty)$
c. $\mathbb{R}^1$
d. (0, 1)

vii. In a McNemar test, if the attribute A is tricategorical and attribute B is biquadricategorical, the degrees of freedom will be

a. 12
b. 7
c. 1
d. 6

viii. Which of the following probability distribution was/were initiated by Indian?

a. Kumaraswamy Distribution
b. Singh-Maddala Distribution
c. Both **a** and **b**
d. Neither **a** nor **b**

Q. 2. Short Answer-type Questions (Attempt any seven) **[14]**

i. Let $Y_i, i = 1,2, \ldots, n$ be a random sample of size n from a $U(0, \theta)$ population. Construct a 95% CI for θ based on a suitable estimator.

ii. Find an unbiased estimator of the mean of an Exponential (θ) population from a sample of size n, based on $X_{(1)}$, the smallest observation in the sample. Is this estimator weakly consistent, strongly consistent, or inconsistent?

iii. Show that the MLE of θ obtained from a sample of size n from an Exponential(θ) population is a biased estimator.

iv. Consider that the number of dead babies in a hospital in a quarter follows a Binomial$(10, p)$ that is used for testing $H_0: p = \frac{1}{2}$ vs. $H_1: p \neq \frac{1}{2}$ .Compute Power of the Critical Regions:

a) {0,10}
b) {0,1,9,10}
c) (0,1,2,8,9,10}

v. State the Neyman-Pearson Lemma. Hence, obtain the MP-test for testing H_0: X~N(0,1) vs H_1: X~Cauchy(0,1)[Standard].

vi. In usual notation, write down the Test Statistic of Bartlett test.

vii. Briefly describe Yate's Correction Table.

viii. Write the algorithm of Non-parametric One-Way ANOVA Test.

ix. Distinguish between Parametric and Non-parametric Test.

(2)

Q. 3. **a.** State the Rao-Blackwell Theorem and the Lehmann-Scheffé Theorem. Consider a random sample of size n from a Poisson distribution X with mean θ. Find the UMVUE of $P_\theta\{X = k\}$, where k is an arbitrary positive integer using either of the above given theorems. **[6]**

b. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from a $U(\theta - 1, \theta + 1)$ population. Consider four estimators: **[6]**

a. $T_1 = \bar{X}$ **b.** $T_2 = X_{(1)} - 1$ **c.** $T_3 = X_{(n)} + 1$ **d.** $T_4 = \frac{T_2 + T_3}{2}$

Which among the above is the best estimator?

[OR]

b. Let $X_1, X_2, \ldots, X_n$ be a random sample from the inverse Gaussian PDF, **[6]**

$$f(x \mid \mu, \lambda) = \left(\frac{\lambda}{2\pi x^3}\right)^{\frac{1}{2}} e^{-\frac{\lambda(x-\mu)^2}{2\mu^2 x}}, x > 0$$

Show that the MLEs of μ and λ are $\bar{X}$ and $\frac{n}{\sum_i\left(\frac{1}{x_i} - \frac{1}{\bar{x}}\right)}$ respectively and that they are independent.

Q. 4. **a.** Heuristically derive the ANOVA F-test and hence, the Two-Independent-Sample-t-Test using Likelihood Ratio Principle. **[6]**

b. Consider the problem of testing $H_0: \theta = \theta_0$ vs $H_1: \theta = \theta_1$, using a random sample of size n from a $U(0, \theta)$ population, $\theta > 0$. Construct all possible Most Powerful size-α Tests and comment on each of them. **[6]**

[OR]

b. Consider a random sample of size n from a $N(\mu, \sigma^2)$ population. Construct UMPU tests of size α for the following hypotheses: **[6]**

i. $H_0: \mu \leq \mu_0$ vs $H_1: \mu > \mu_0$, σ^2 unknown

ii. $H_0: \mu = \mu_0$ vs $H_1: \mu \neq \mu_0$, σ^2 unknown

iii. $H_0: \sigma^2 \geq {\sigma_0}^2$ vs $H_1: \sigma^2 < {\sigma_0}^2$, μ unknown

Q. 5. **a.** Given for a large sample data of size 50, we conducted a run test that gives mean 25.64 and standard deviation 3.45. Find the sample sizes of two groups. **[6]**

b. Discuss Mann-Whitney-U Test for two samples. **[6]**

[OR]

b. Write three application of Chi-Square Test with detail. **[6]**

[Turn Over]

CP-T-07

Q. 6. **a.** Given in usual notation, $R_1 = 11, R_2 = 13, R_3 = 18, F_r = 3.71$. Find number of subjects and number of groups. **[6]**

b. Three sides of an equilateral triangle were measured by 5 persons with the following results. Are there any significant differences between the measurements of the persons at $100(1-\alpha)\%$ confidence level? Use Jockhnere-Terpestra Test to give your conclusion. **[6]**

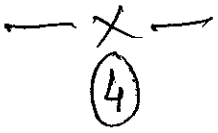
Side/Person	*A*	*B*	*C*	*D*	*E*
a	5.44	5.41	5.43	5.42	5.43
b	5.43	5.41	5.42	5.43	5.44
c	5.45	5.42	5.43	5.43	5.44

[OR]

b. The following data represent lifetime (hours) of batteries for two different brands **[6]**

Brand A	40	30	40	45	55	30
Brand B	50	50	45	55	60	40

Use appropriate test to conclude that the two samples come from the same population.



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