(85) SEAT No._____ No. of Printed Pages : 3

SARDAR PATEL UNIVERSITY

M.Sc. (Applied Statistics), Semester I

PS01CAST21: (Statistical Distributions and their Applications)

22 October 2018, Monday

Time: 10:00 AM– 01:00PM Marks: 70

Q.1) Attempt all Multiple Choice Questions [8]

i) Which of the following is a particular parameterization of the Beta distribution of the second kind?

a) Exponential Distribution
b) Student's t-Distribution
c) F-Distribution
d) Gamma Distribution

ii) Which of these distributions is referred to when testing for equality of a pair of means?

a) Chi-Squared Distribution
b) F-Distribution
c) Student's t-Distribution
d) Wishart Distribution

iii) Which of these distributions would you use to fit the flight arrival/departure rate at an airport?

a) Poisson Distribution
b) Exponential Distribution
c) Normal Distribution
d) Bernoulli Distribution

iv) Which of these is a real-life application of the Binomial Distribution?

a) To fit the number of road accidents in a busy road
b) To fit the amount of money collected at a toll booth
c) To fit the frequency of a particular sound wave
d) To fit the number of cars parked in a particular parking space

v) If $\underline{X} = [X_1 \quad X_2 \quad X_3 \quad \dots \quad X_p]'$ follows a p-variate Normal Distribution with parameters $\underline{\mu}$, and covariance matrix Σ, then

a) The marginal distributions of X_i, i = 1,2,…,p, follow Univariate Normal Distributions
b) Any sub-vector of $\underline{X}$ of order k×1 follows a k- variate Normal Distribution
c) Σ is a symmetric positive semi-definite square matrix of order p.
d) All of the above

vi If $\underline{X} = [X_1 \quad X_2 \quad X_3 \quad \dots \quad X_p]'$ follows a p-variate Normal Distribution with parameters $\underline{0}$, and covariance matrix $\Sigma = I$, then $\underline{X}'\underline{X}$ follows

a) Wishart Distribution with p d.f.
b) Chi-Squared Distribution with p d.f.
c) t-Distribution with p d. f.
d) F-Distribution with p d.f.

vii) Which of these is NOT true for the Cauchy Distribution?

a) It is basically a transformation taken on a $U(-\pi, \pi)$ variate
b) Its Moment Generating Function does not exist
c) Its Characteristic function exists
d) Cannot be randomly generated using Probability Integral Transformation

viii) Another name of double exponential distribution is ….

a) Truncated Exponential
b) Exp. with location and scale parameter
c) Laplace Distribution
d) Two parameter Weibull

Q.2) Attempt any seven. [14]

i) Derive the Hazard Rate for an Exponential Variate with mean 10.

ii) Write down the m.g.f of the three-parameter Gamma Distribution. Use it to derive its mean and variance.

iii) Write down the p.d.f of the Beta distribution. Derive its r^{th} order moment.

iv) Write down all the important properties of the Univariate Normal Distribution.

v) Show that the sum of any number of Bernoulli variates gives a Binomial variate.

vi) Generate any 3 values of a one-parameter Exponential variate with variance 16, using Probability Integral Transformation.

vii) The p.g.f of a random variable X is given as $P_X(t) = [(1-p)+tp]^n; t \geq 0, 0 \leq p \leq 1, n = 0,1,2, \ldots..$ Obtain the value of $P(X \leq 2)$, and identify the distribution of X.

viii) Derive the m.g.f of the Geometric Distribution and hence obtain its mean and variance. Also write down its applications.

ix) Write a short note on the Log-Normal Distribution.

x) If a random variate $X \sim U\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then find the distribution of $Y = tan\ X$.

Q.3)a) Derive the p.d.f of the F-Distribution. Write down its applications. [6]

b) Derive the m.g.f of a three-parameter Weibull variate. Hence, obtain its mean, and variance. [6]

== OR ==

b) Derive the Characteristic Function of the Cauchy Distribution. State its relation with t-distribution having 1 degree of freedom. [6]

Q.4)a) Derive the p.d.f. of the central Chi-squared Distribution. Also, write down its properties. [6]

b) Derive the Distribution Function of the Laplace Distribution. [6]

== OR ==

b) A r.v X has Poisson distribution with mean λ, while the conditional distribution of Y for given $X = x$ $(x = 0,1,2, \ldots)$ is a Poisson variate with mean μx. Obtain the p.g.f of Y [6]

Q.4)a) Derive the m.g.f. of the Pareto Distribution. [6]

b) Obtain the Hazard Rates of: [6]

i. Pareto Distribution

ii. Weibull Distribution (3-parameter)

iii. Logistic Distribution

==OR==

b) A R.V. X has a lognormal distribution with mean $e^{\mu+\frac{\sigma^2}{2}}$. Derive its r^{th} order moment and, hence, derive Skewness and Kurtosis. [6]

Q.6)a) Obtain the mean and variance of the Hypergeometric Distribution. [6]

b) Write down the p.d.f of the non-singular Multinomial Distribution, obtain its m.g.f, and hence, obtain its mean and variance. [6]

==OR==

b) Write down the characteristic function of the Multivariate Normal Distribution. Hence show that any linear combination of a Multivariate Normal vector is also Multivariate Normal. [6]

—X—