

Time: 10:30 A.M to 01:30 P.M

Total Marks: 70

Note: Write answers of both the sections in separate answer sheets.

Section - I

Q.1 (a) Attempt any *two*:

[06]

(i) Solve the system of equations

$$2x_1 + x_2 + x_3 = 6$$

$$x_2 + 2x_3 = 7$$

$$2x_3 = 6$$

by using back substitution method.

(ii) Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$. Find $A^2 - 4A$.

(iii) Convert $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 1 & 2 \end{bmatrix}$ into upper triangular form.

[06]

(b) Attempt any *one*:

(i) Find the rank of $\begin{bmatrix} 2 & 1 & 10 & 11 & 1 & 16 \\ 3 & 2 & 17 & 20 & 2 & 28 \\ 1 & 1 & 7 & 9 & 3 & 22 \\ 2 & 1 & 10 & 11 & 5 & 36 \end{bmatrix}$.

(ii) Find basic solutions of the following system of equations.

$$x_1 + x_3 + 3x_4 + 4x_5 + 3x_7 = 0$$

$$x_2 + 4x_3 + 7x_4 + 2x_5 + 4x_7 = 0$$

$$x_6 + 5x_7 = 0$$

[06]

Q.2 (a) Attempt any *two*:

[06]

(i) Solve $a_{n+2} - 11a_{n+1} + 24a_n = 0$.

(ii) Find characteristic roots of $a_{n+3} - 3a_{n+2} + 3a_{n+1} - a_n = 0$.

(iii) Solve $4a_{n+2} - 4a_{n+1} + a_n = 0$.

[06]

(b) Attempt any *one*:

(i) Solve $a_{n+2} - 7a_{n+1} + 12a_n = 5(10^n)$.

(ii) Solve $a_{n+1} - 7a_n = (2+n)3^n$.

[06]

Q.3 (a) Attempt any *two*:

(i) Define contradiction and give one example of it.

(ii) Using truth table show that $\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$.

(iii) Using mathematical induction show that $2^n < n!$, $n = 4, 5, 6, \dots$

[05]

(b) Attempt any *one*:

(i) Find the principal disjunctive normal form of $\neg P \vee Q$.

(ii) For atomic variables P, Q, R , write all maxterms and minterms.

Section – II

- Q.4**
[A] **Answer the following questions:** [9]
(i) Define the term graph. Explain components of graph with example.
(ii) Explain subgraphs with suitable example.
(iii) Explain the concept of cut-sets.
- [B] Write a note on incidence matrix. [5]
- Q.5**
[A] Prove: A tree with n vertices has $n-1$ edges. [5]
[B] Define spanning tree. What do you mean by rank and nullity of graph? [5]
Explain a method of finding all spanning trees in a graph.
- Q.6**
[A] Prove: The number of vertices of odd degree in a graph is always even. [5]
[B] Answer the following questions: [6]
(i) Differentiate between connected graph and complete graph.
(ii) Draw binary trees of 11 vertices with possible maximum and minimum height.
(iii) Draw and explain minimally connected graph.
