[A-22]

SEAT No.________

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SARDAR PATEL UNIVERSITY

B.Sc.(SEMESTER - III)(2010-BATCH) EXAMINATION (NC)

SC

Wednesday , 28^{th} November,2018

US03EMTH05 (CALCULUS AND ALGEBRA - I)

Time : 2:00 p.m. to 4:00 p.m. **Maximum Marks : 70**

Que.1 Attempt the following. 10

(1) $\log \infty = \ldots\ldots$

(a) 1 (b) 0 (c) ∞ (d) $-\infty$

(2) $\lim_{x \to 0} \frac{\log(\cos x)}{\sin x}$ is of the form ....

(a) $\frac{0}{0}$ (b) $\infty - \infty$ (c) $\frac{\infty}{\infty}$ (d) $0 \cdot \infty$

(3) If $f(x) = \sin x$ then $f_{xx} = \ldots\ldots$

(a) $\sin x$ (b) $\cos x$ (c) $-\cos x$ (d) $-\sin x$

(4) $\frac{\partial^2 f}{\partial x^2} + \left[\frac{\partial f}{\partial x}\right]^4$ is of degree.....

(a) 2 (b) 1 (c) 6 (d) 4

(5) If $u = x^3y - xy^3$ then $u_x = \ldots\ldots$

(a) $3x^2y - y^3$ (b) $3y^2x - y^3$ (c) $3x^2y + y^3$ (d) $x^4y + x^2y^3$

(6) If $A = \begin{bmatrix} 9 & i \\ 7 & 2i \end{bmatrix}$ then $A^\theta = \ldots\ldots$

(a) $\begin{bmatrix} 9 & i \\ 7 & 2i \end{bmatrix}$ (b) $\begin{bmatrix} 9 & -1 \\ 7 & -2i \end{bmatrix}$ (c) $\begin{bmatrix} 9 & 7 \\ -i & -2i \end{bmatrix}$ (d) $\begin{bmatrix} 9 & 7 \\ i & 2i \end{bmatrix}$

(7) If C = A + B , where A $= \begin{bmatrix} -1 & 6 \\ i & 17 \end{bmatrix}$ and B $= \begin{bmatrix} -i & 6 \\ i & 1 \end{bmatrix}$ then C = ...

(a) $\begin{bmatrix} -1 & 6 \\ i & 17 \end{bmatrix}$ (b) $\begin{bmatrix} -i & 6 \\ i & 1 \end{bmatrix}$ (c) $\begin{bmatrix} -(1+i) & 12 \\ 2i & 18 \end{bmatrix}$ (d) $\begin{bmatrix} 1+i & 6 \\ 2i & 17 \end{bmatrix}$

(8) If A $= \begin{bmatrix} 2 & 0 & 4 \\ 6 & i & 8 \\ 5 & 6 & 9 \end{bmatrix}$ then entry $a_{23} = \ldots\ldots$

(a) 6 (b) 8 (c) 0 (d) i

(9) If A and B are commutative to each other then $(A+B)^2 = \ldots\ldots\ldots$

(a) $A^2 - 2AB + B^2$ (b) $A^2 + B^2$ (c) $A^2 + 2AB + B^2$ (d) $A^2 + AB + BA + B^2$

(10) If $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ then $A^2 = \ldots\ldots\ldots$

(a) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

Que.2 Attempt the following.(**Any Ten**) 20

(1) Evaluate $\lim_{x\to 0} \dfrac{\log(\sin x)}{\cot x}$.

(2) Evaluate $\lim_{x\to 2} \dfrac{\tan(x^2-4)}{x-2}$.

(3) Evaluate $\lim_{x\to 0} \dfrac{x-\tan x}{x^3}$.

(4) For $u = x^3 - 3xy^2$ prove that $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$.

(5) If $u = \sqrt{x^2+y^2}$ then find $\dfrac{\partial^2 u}{\partial x^2} - \dfrac{\partial^2 u}{\partial y^2}$.

(6) Find $\dfrac{\partial u}{\partial x}$ and $\dfrac{\partial u}{\partial y}$ for $u = \log(x^2+y^2)$.

(7) Define Transpose of matrix and lower Triangular matrix with example.

(8) If A and B are both symmetric then prove that AB is symmetric iff A and B commute.

(9) If A is Hermitian then prove that BAB^θ is Hermitian.

(10) If $A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$ then find $A^2 + 4A$.

(11) If $A = \begin{pmatrix} 2 & -3 \\ -1 & 4 \end{pmatrix}$ then find characteristic matrix and characteristic equation of A.

(12) If $A = \begin{pmatrix} 1 & 2 & 5 \\ -2 & 6 & -8 \\ 5 & 8 & 7 \end{pmatrix}$ then find $|A'|$.

Que.3 (a) Evaluate $\lim_{x\to 0} \dfrac{e^x - e^{-x} - 2\log(1+x)}{x \sin x}$. 5

(b) Evaluate $\lim_{x\to 0} \dfrac{\tan x \tan^{-1} x - x^2}{x^6}$. 5

OR

Que.3 (c) Evaluate $\lim_{x\to a} (2 - \frac{x}{a})^{\tan(\frac{\pi x}{2a})}$. 5

(d) Find a,b and c for which $\lim_{x\to 0} \dfrac{ae^x - 2b\cos x + 3ce^{-x}}{x \sin x} = 2$. 5

Que.4 (a) State and prove Euler's theorem for function of two variables . 5

(b) If $z = f(x,y)$, $x = r\cos\theta, y = r\sin\theta$ then prove that
$$\left[\frac{\partial z}{\partial x}\right]^2 + \left[\frac{\partial z}{\partial y}\right]^2 = \left[\frac{\partial z}{\partial r}\right]^2 + \frac{1}{r^2}\left[\frac{\partial z}{\partial \theta}\right]^2$$

OR

Que.4 (c) If $H = f(2x-3y, 3y-4z, 4z-2x)$ then prove that $\dfrac{1}{2}\dfrac{\partial H}{\partial x} + \dfrac{1}{3}\dfrac{\partial H}{\partial y} + \dfrac{1}{4}\dfrac{\partial H}{\partial z} = 0$ 5

(d) Verify Euler's theorem and find $x^2\dfrac{\partial^2 z}{\partial x^2} + 2xy\dfrac{\partial^2 z}{\partial x \partial y} + y^2\dfrac{\partial^2 z}{\partial y^2}$ for $z = x^n \log \frac{y}{x}$. 5

Que.5 (a) Prove that Every square matrix can be expressed in one and only one way as the sum of a symmetric and skew symmetric matrix. 5

(b) If $A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & -2 & 1 \\ -2 & 2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & -2 \\ 1 & -2 & -1 \\ 1 & 1 & 2 \end{pmatrix}$ then prove that $(AB)^T = B^T A^T$. 5

OR

Que.5 (c) Prove that Every square matrix can be expressed in one and only one way as $P + iQ$, where P and Q are Hermitian matrices. 5

(d) IF $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 3 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & -2 \\ 1 & -5 \\ 4 & 3 \end{bmatrix}$ then find $A^T B$, $(B^T A)^T$. 5

Que.6 (a) State and prove Cayley-Hamilton theorem. 5

(b) Verify Cayley-Hamilton theorem for $A = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$. 5

OR

Que.6 (c) If $A = \begin{pmatrix} 3+4i & 4-5i \\ 5-6i & 2+i \end{pmatrix}$ then find $|A|$ and $|A'|$ also prove that $|\overline{A}| = \overline{|A|}$. 5

(d) If $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ then find out the values of α , β such that $(\alpha I + \beta A)^2 = A$. 5

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