SE

[2/A-9]

SEAT No.________

No of printed pages : 3

SARDAR PATEL UNIVERSITY
B.Sc.(SEMESTER - III)
Thursday , 6^{th} December,2018

US03EMTH05 (CALCULUS AND ALGEBRA - I)

Time : 2:00 p.m. to 4:00 p.m. Maximum Marks : 70

Que.1 Attempt the following. 10

(1) $\lim_{x\to 1} \dfrac{\log(4-4x^2)}{\log(2-2x)}$ is of the form ....

(a) $\dfrac{0}{0}$ (b) $\infty - \infty$ (c) $\dfrac{\infty}{\infty}$ (d) $0\cdot\infty$

(2) $\log 0 =$ ....

(a) 0 (b) $-\infty$ (c) 1 (d) -1

(3) If $u = x^3y - xy^3$ then $u_x =$ ....

(a) $3x^2y - y^3$ (b) $3y^2x - y^3$ (c) $3x^2y + y^3$ (d) $x^4y + x^2y^3$

(4) $F = \dfrac{xy}{x+y}$ is homogeneous function of degree ....

(a) 0 (b) 1 (c) 2 (d) None

(5) If $f(x,y) = xy$ then $f_{xy}=$ ....

(a) 0 (b) y (c) xy (d) 1

(6) If $A = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$ then $A' =$ ....

(a) $\begin{bmatrix} -3 & 0 \\ -2 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}$

(7) The matrix $B = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 6 & 2i \\ 0 & 0 & i \end{bmatrix}$ is .... matrix .

(a) Upper triangular (b) Lower triangular (c) Diagonal (d) Symmetric

(8) If $A = \begin{bmatrix} 9 & i \\ 7 & 2i \end{bmatrix}$ then $A^\theta =$ ....

(a) $\begin{bmatrix} 9 & i \\ 7 & 2i \end{bmatrix}$ (b) $\begin{bmatrix} 9 & -1 \\ 7 & -2i \end{bmatrix}$ (c) $\begin{bmatrix} 9 & 7 \\ -i & -2i \end{bmatrix}$ (d) $\begin{bmatrix} 9 & 7 \\ i & 2i \end{bmatrix}$

(9) If $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ then $A^2 =$ ....

(a) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

(10) If A and B are not commutative to each other then $(A+B)^2 =$ ....

(a) $A^2 - 2AB + B^2$ (b) $A^2 + B^2$ (c) $A^2 + 2AB + B^2$ (d) $A^2 + AB + BA + B^2$

P.T.O.

Que.2 Attempt the following.**(Any Ten)** 20

(1) Evaluate $\lim_{x\to 0} \dfrac{\log(\sin 2x)}{\log(\sin x)}$.

(2) Evaluate $\lim_{x\to 2} \dfrac{\tan(x^2-4)}{x-2}$.

(3) Evaluate $\lim_{x\to 0} \left[\dfrac{\tan x}{x}\right]^{\dfrac{5}{3x^2}}$.

(4) For $u = x^3 + 3xy^2$ find $\dfrac{\partial^2 u}{\partial x^2} - \dfrac{\partial^2 u}{\partial y^2}$.

(5) If $u = \sin^{-1}\left(\frac{x^2y^2}{x+y}\right)$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3\tan u$.

(6) Find $\dfrac{\partial u}{\partial x}$ and $\dfrac{\partial u}{\partial y}$ for $u = \frac{1}{\sqrt{x^2+y^2}}$.

(7) Prove that $(BA)^\theta = A^\theta B^\theta$.

(8) For $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ Show that $A'A = I$.

(9) If $A = \begin{pmatrix} 2 & 1 & 1 \\ -1 & -1 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -3 & 4 \\ -3 & 1 & -2 \end{pmatrix}$ then find $5A + 3B$.

(10) If $A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$ then find characteristic matrix and characteristic equation of A.

(11) If $A = \begin{pmatrix} 13 & 16 & 19 \\ 14 & 17 & 20 \\ 15 & 18 & 21 \end{pmatrix}$ then find $|A|$.

(12) If $A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$ then find $A^2 - 4A$.

Que.3 (a) Find a,b and c for which $\lim_{x\to 0} \dfrac{ae^x - 2b\cos x + 3ce^{-x}}{x\sin x} = 2$. 5

(b) Evaluate $\lim_{x\to 0} \dfrac{\log(\log(1-3x^2))}{\log(\log(\cos 2x))}$. 5

OR

Que.3 (c) Find a,b and c so that $\lim_{x\to 0} \dfrac{a\sin x - bx + cx^2 + x^3}{2x^2\log(1+x) - 2x^3 + x^4}$ be finite. Also determine the limit. 5

(d) Evaluate $\lim_{x\to 0} \dfrac{e^x + \log(1-x) - 1}{\tan x - x}$. 5

Que.4 (a) State and prove Euler's theorem for function of three variables . 5

(b) Verify Euler's theorem and find $x^2\dfrac{\partial^2 z}{\partial x^2} + 2xy\dfrac{\partial^2 z}{\partial x \partial y} + y^2\dfrac{\partial^2 z}{\partial y^2}$ for $z = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$. 5

OR

Que.4 (c) Find $\dfrac{dz}{dt}$ when $z = \sin^{-1}(x-y), x = 3t, y = 4t^3$. Also verify by the direct substitution. 4

(d) If $z = f(x,y)$, $x = r\cos\theta, y = r\sin\theta$ then prove that
$\left[\dfrac{\partial z}{\partial x}\right]^2 + \left[\dfrac{\partial z}{\partial y}\right]^2 = \left[\dfrac{\partial z}{\partial r}\right]^2 + \frac{1}{r^2}\left[\dfrac{\partial z}{\partial \theta}\right]^2$. 6

P.T.O.

Que.5 (a) Prove that Every square matrix can be expressed in one and only one way as the sum of a symmetric and skew symmetric matrix. 5

(b) If $A = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 2 \\ -2 & 2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & -2 \\ 3 & 2 & -1 \\ -1 & -2 & 0 \end{pmatrix}$ then prove that $(BA)^T = A^T B^T$. 5

OR

Que.5 (c) Prove that Every square matrix can be expressed in one and only one way as $P + iQ$, where P and Q are Hermitian matrices. 5

(d) If $A = \begin{pmatrix} 2+i & 3-i & 4+5i \\ 1+3i & 2i & 5-6i \\ 3+i & 6-5i & 1+i \end{pmatrix}$ then find $(\overline{A})'$ and AA^{θ}. 5

Que.6 (a) State and prove Cayley-Hamilton theorem. 5

(b) For $A = \begin{pmatrix} 2 & 2 \\ 3 & -4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 7 \\ 5 & 3 \end{pmatrix}$, $C = \begin{pmatrix} -1 & 5 \\ 3 & 2 \end{pmatrix}$, prove that $(AC)B = A(CB)$ 5

OR

Que.6 (c) Verify Cayley-Hamilton theorem for $A = \begin{pmatrix} 3 & 1 \\ -2 & 2 \end{pmatrix}$. 5

(d) If $A = \begin{pmatrix} 5 & 7 & 3 \\ 1 & 5 & 2 \\ 3 & 2 & 1 \end{pmatrix}$ then find $A^3 - 11A^2 + 15A$. 5

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