[41/A-21] SEAT No.________ **No. of Printed Pages : 3**

SARDAR PATEL UNIVERSITY

B.Sc. Sem-III

Subject: Mathematics

Course No: US03EMTH01 (Calculus)

Date: 28/11/2018 (Wednesday) Time: 2.00PM-4.00PM

Total marks: 70

Q-1 Choose the correct answer from the options given below. **[10]**

1. $\nabla \times (\nabla f) =$ ______

[a] 0 [b] 1 [c] $\bar{0}$ [d] None

2. Integral $\int_1^2 x^2 dx$ is ________

[a] Improper integral of 1^{st} kind [b] Proper integral

[c] Improper integral [d] None

3. Relation between Beta and Gamma function is $\beta(m, n) =$ ________

[a] $\frac{\Gamma mn}{\Gamma m+n}$ [b] $\frac{\Gamma m + \Gamma n}{\Gamma m+n}$ [c] $\frac{\Gamma m \Gamma n}{\Gamma m+n}$ [d] None

4. $\nabla \cdot (\nabla \times \bar{v}) =$ ________

[a] 1 [b] -1 [c] 0 [d] None

5. $\Gamma n =$ ________

[a] n! [b] n-1 [c] (n-1)! [d] None

6. If $f(x)$ is odd then $\int_{-c}^{c} f(x)dx =$ ______

[a] 1 [b] -1 [c] 0 [d] None

7. $\nabla \cdot (f \nabla g) =$ ________

[a] $\nabla f \cdot \nabla g$ [b] $f\nabla^2 g$ [c] $f\nabla^2 g + \nabla f \cdot \nabla g$ [d] None

8. Constant function is a ________ function.

[a] Even [b] Odd [c] Periodic [d] None

9. Integral $\int_1^\infty x dx$ is ______

[a] Proper integral [b] Improper integral of 2^{nd} kind

[c] Improper integral of 1^{st} kind [d] None

10. $\Gamma n =$ ________

[a] $\int_0^\infty x^{n-1} e^x dx$ [b] $\int_0^1 x^{n-1} e^{-x} dx$ [c] $\int_0^\infty x^{n-1} e^{-x} dx$ [d] None

(PTO)

Q-2 **Answer briefly. (Attempt any ten)** **[20]**

1. Prove that $\beta(m,n)=\beta(n,m)$.
2. Show that product of even and odd function is odd.
3. Define the Convergence of Integral.
4. Define: (1) Beta function, (2) Gamma function.
5. Evaluate: $\int_0^{\infty} e^{-x^2}\,dx$.
6. Define Periodic function and give its example.
7. Prove that $\overline{\nabla}(f+g)=\overline{\nabla}f+\overline{\nabla}g$.
8. Define curl of a vector field.
9. Prove that $\left\lceil\frac{1}{2}\right. = \sqrt{\pi}$.
10. Define Fourier Series.
11. Evaluate: $\int_0^{\pi/2} \frac{\sqrt{x}}{\sin x}\,dx$.
12. Find the Fourier co-efficient a_0 for $f(x)=1$, $\frac{-\pi}{2}<x<\frac{\pi}{2}$.

Q-3

(a) Prove that the integral $\int_a^b \frac{dx}{(b-x)^{\mu}}$ is convergent if and only if $\mu<1$. **[6]**

(b) Evaluate: (1) $\int_{-\infty}^{\infty} \frac{1}{x^2}\,dx$, (2) $\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$ **[4]**

OR

Q-3

(c) Prove that the integral $\int_a^{\infty} \frac{1}{x^{\mu}}\,dx$ (a>0), is convergent iff $\mu>1$. **[6]**

(d) Examine the convergence of $\int_1^{\infty} \frac{dx}{x^{1/2}(1+x)^{1/2}}$. **[4]**

Q-4

(a) Prove that $\beta(m,n)=\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}\,dx$. **[6]**

(b) Evaluate $\int_0^{\pi/2} \sin^5\theta \cos^2\theta \, d\theta$. [4]

OR

Q-4

(c) Evaluate $\int_0^1 x^5(1-x^3)^{10}dx$. [6]

(d) Prove that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta$. [4]

Q-5

(a) Prove that (1) $\nabla(fg) = f\nabla g + g\nabla f$, (2) $\nabla(\frac{f}{g}) = \frac{g\nabla f - f\nabla g}{g^2}$. [5]

(b) Find the directional derivative of $f(x,y,z) = 4xz^3 - 3x^2y^2z$ at point (2, -1, 2) in the direction of $\bar{a} = \bar{i} - 2\bar{k}$. [5]

Q-5

OR

(c) Find gradient of function $f(x,y,z) = (x^2+y^2+z^2)^2$ at (1,2,3). [5]

(d) Prove that $\nabla \cdot (\nabla \times \bar{v}) = 0$ [5]

Q-6 Find Euler's constant a_n, b_n for Fourier series of a function f(x) over $[-\pi,\pi]$. **[10]**

OR

Q-6 Find the Fourier series of the following: **[10]**
(1) $f(x) = x^2, -\pi < x < \pi$, (2) $f(x) = x, -\pi < x < \pi$

—x—