[5/A-10]

SEAT No._____

No. of Printed Pages : 3

Sardar Patel University, Vallabh Vidyanagar

B.Sc. [Semester-III] Examinations : 2018-19

Subject : Mathematics US03CMTH02 Max. Marks : 70

Numerical Analysis

Date: 05/12/2018, Wednesday Timing: 02.00 pm - 05.00 pm

Q: 1. Answer the following by choosing correct answers from given choices. 10

[1] If $f(x) = 0$ is expressed as $x = \phi(x)$ then for approximation of root of $f(x) = 0$ using Iteration method, one of the necessary conditions for the convergence of a sequence of approximations is that

[A] $|\phi(x)| > 1$ [B] $|\phi(x)| < 1$ [C] $|\phi'(x)| > 1$ [D] $|\phi'(x)| < 1$

[2] In usual notations, the formula $\xi = x_{i+1} - \dfrac{(\Delta x_i)^2}{\Delta^2 x_{i-1}}$ is used by the method of

[A] False position [B] Bisection [C] Iteration [D] Aitken's Δ^2-Process

[3] For approximation of a root of an equation, intersection of a chord joining end points of graph of a function in an interval and the X-axis is used in

[A] False position method [B] Bisection method

[C] Iteration method [D] Aitkin's Δ^2-Process

[4] $\delta_{\frac{7}{2}}$ is given by

[A] $y_4 - y_3$ [B] $y_3 - y_4$ [C] $y_7 - y_2$ [D] $y_2 - y_7$

[5] Which of the follwing is true?

[A] $\Delta y_5 = \nabla y_4$ [B] $\Delta y_5 = \nabla y_5$ [C] $\Delta y_4 = \nabla y_5$ [D] $\Delta y_6 = \nabla y_5$

[6] If $y_5 = 4$, and $y_{15} = 10$ then $E^5 y_{10} =$

[A] 5 [B] 10 [C] 15 [D] 20

[7] The divided differences are

[A] not dependent on their arguments

[B] symetrical in their arguments

[C] not symetrical in their arguments

[D] none

[8] For the given data

x	$x_0 = 2$	$x_1 = 6$	$x_2 = 10$	$x_3 = 14$
y	15	20	32	50

$[x_1\ x_2] =$

[A] 1 [B] 2 [C] 3 [D] none

[9] Runge-Kutta method is used for finding a numeric

[A] integral [B] derivative [C] solution of a differential equation [D] none

[10] For using Simpson's $\frac{1}{3}$ rule it is required that the number of sub-intervals be

[A] even [B] odd [C] a multiple of 3 [D] a multiple of 8

(P.T.O.)

Q: 2. Answer any TEN of the following. 20

[1] Find an interval containing an initial approximation of $\sin x = \cos x$

[2] Find first approximation of a root of $x^3 + 8x - 7 = 0$ using bisection method

[3] Find a real root of $x^3 - 3x + 5 = 0$, correct upto three decimal places, by Newton-Raphson method

[4] Prove that $\Delta - \nabla = \Delta\nabla$

[5] If $E^{10}y_1 = 20$ then find $E^5y_6 + E^6y_5$

[6] Prove that $e^{hD} = E$

[7] If $y_1 = 4,\ y_3 = 12,\ y_4 = 19$ and $y_x = 7$ find x. Write the formula you use and also give it's name

[8] Using Langrange's interpolation formula, find the form of the function $y(x)$ from the following table

x	0	1	3	4
y	-12	0	12	24

[9] In usual notations prove that $[x_0, x_1, x_2, x_3, ..., x_n] = \dfrac{1}{h^n.n!}\Delta^n y_0$

[10] Using Trapezoidal rule find $\int_0^5 \frac{1}{x+1}dx$, with subintervals of length 1 unit.

[11] Discuss Euler's method for solving a differential equation.

[12] Using Trapezoidal rule find $\int_0^3 \cos x dx$, with 3 subintervals of equal lengths.

Q: 3 [A] Discuss the Aitken's Δ^2-Process for approximation of a real root of an equation. 5

[B] Using Bisection method find a real root of the equation $x^3 - x - 4 = 0$ correct upto three decimal palaces 5

OR

Q: 3 [A] Discuss the False Position method for approximation 5

[B] Find a real root of $\cos x = 3x - 1$ by iteration method correct upto three decimal places 5

Q: 4 [A] Derive Newton's Forward Difference interpolation formula for equally spaced values of arguments. 5

[B] Using Gauss's forward interpolation formula find $f(32)$, given that $f(25) = 0.2707,\ f(30) = 0.3027,\ f(35) = 0.3386,\ f(40) = 0.3794$ 5

OR

Q: 4 [A] Use Stirling's formula to find u_{32}, given that

$$u_{20} = 14.035,\ u_{25} = 13.674,\ u_{30} = 13.257,\ u_{35} = 12.734,\ u_{40} = 12.089 \text{ and } u_{45} = 11.309$$

5

[B] Let $y = g(x)$ be a function such that

$$g(20) = 2854,\ g(24) = 3162,\ g(28) = 3544,\ g(32) = 3992$$

Use Everett's formula to obtain $g(25)$. 5

Q: 5 [A] Derive Newton's divided difference formula 5

[B] The following table of values of x and y is given :

x	0	1	2	3	4	5	6
y	6.9897	7.4036	7.7815	8.1291	8.4510	8.7506	9.0309

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when $x = 6$ 5

OR

Q: 5 [A] Discuss the method of succesive apprximation for inverse interpolation. 5

[B] Using Langrange's interpolation formula express the following function as a sum of partial fractions

$$\frac{x^2 + x - 3}{x^3 - 2x^2 - x + 2}$$

5

Q: 6 [A] Derive the formula of Simpson's $\frac{3}{8}$-rule for numerical integration. 5

[B] Using Romberg's method, compute $I = \int_0^1 \frac{1}{1+x}\,dx$, correct upto three decimal places 5

OR

Q: 6 [A] Describe Picard's method of successive approximation and use it to approximate y when $x = 0.2$, given that $y(0) = 1$ and $\frac{dy}{dx} = x - y$, correct upto three decimal places 10

—X—

[3]