[A-24]

SEAT No.________

No. of printed page : 3

SARDAR PATEL UNIVERSITY

B.Sc.(SEMESTER - III)(2010 - Batch) EXAMINATION (NC)

Tuesday , 20^{th} November, 2018

US03CMTH01 (ADVANCED CALCULUS)

Time : 02.00 p.m. to 05.00 p.m. **Maximum Marks:70**

Que.1 Fill in the blanks. 10

(1) Area of region R is given by A = ...

(a) $\iint_R dxdy$ (b) $\iint_C dxdy$ (c) $\iint_R f(x,y)\,dxdy$ (d) $\iint_R dxdydz$

(2) $\int_0^1\int_0^3 dydx =$ ....

(a) 1 (b) 0 (c) 3 (d) 2

(3) In double integral , Total mass M of density 1 over region $0 \le x \le 3$; $0 \le y \le 2$ is ....

(a) 1 (b) 6 (c) 2 (d) 4

(4) $\int_C [fdx + gdy + hdz]$ is independent of path iff $fdx + gdy + hdz$ is ....

(a) 0 (b) not exact (c) 1 (d) exact

(5) Vector form of Green's theorem is $\iint_R (\nabla \times \overline{v}) \cdot \overline{k}\ dxdy =$ ....

(a) $\int_C \overline{v} \cdot \overline{u}\ ds$ (b) $\int_C \overline{v} \cdot \overline{u}\ dx$ (c) $\int_C \overline{v} \cdot \overline{n}\ ds$ (d) $\int_C \overline{v} \times \overline{n}\ ds$

(6) Area of plane region in Cartesian form is given by A = ...

(a) $\frac{1}{2}\int_C [x\ dx + y\ dy]$ (b) $\frac{1}{2}\int_C [x\ dy - y\ dx]$ (c) $\frac{1}{2}\int_C [x\ dy + y\ dx]$ (d) $\frac{1}{2}\int_C [x\ dx - y\ dy]$

(7) Area of a surface $\overline{r}(u,v)$ is A = ...

(a) $\iint_R \sqrt{EG - F^2}\, dxdy$ (b) $\iint_R \sqrt{EG + F^2}\, dxdy$ (c) $\iint_R \sqrt{EG - F^2}\, dudv$ (d) $\iint_R \sqrt{EG - F}\, dudv$

(8) Parametric form of the plane $y = 2x$ is $\overline{r} =$................

(a) $u\overline{i} + v\overline{j} + u\overline{k}$ (b) $u\overline{i} + u\overline{j} + v\overline{k}$ (c) $u\overline{i} + 2u\overline{j} + v\overline{k}$ (d) $u\overline{i} + \overline{j} + v\overline{k}$

(9) A function $f(x, y, z)$ is said to be harmonic if $\nabla^2 f =$ ....

(a) 1 (b) 2 (c) −1 (d) 0

(10) $\int_0^2\int_0^2\int_0^3 dxdydz =$ ....

(a) 12 (b) 6 (c) 3 (d) 2

(P.T.O.)

Que.2 Attempt the following (Any Six) 12

(1) Evaluate $\int_0^1 \int_y^{2y} (1 + x^2 + y^2)\, dxdy$.

(2) Evaluate $\int_0^1 \int_{y^2}^{y} (1 - xy)\, dxdy$.

(3) Change the order of integration in $\int_0^c \int_0^y f(x, y)\, dydx$.

(4) Show the the form under integral sign $\int_{(2,0,0)}^{(1,2,3)} [xdx + ydy + zdz]$ is exact.

(5) State Green's theorem in vector form .

(6) Obtain first fundamental form of a surface in polar form .

(7) Prove second form of Green 's theorem .

(8) Evaluate $\int_C \overline{V} \cdot \overline{t}\, ds$, by using Stoke's theorem , where $\overline{V} = z\overline{i} + x\overline{j}$ and $S : 0 \leq x \leq 1$, $0 \leq y \leq 1$, $z = 1$.

Que.3 (a) Transform $\iint_R (x + y)^3\, dxdy$ in uv - plane by taking $x + y = u$, $x - 2y = v$. Then evaluate it , where R : Parallelogram with vertices $(1, 0), (0, 1), (3, 1), (2, 2)$. 8

OR

Que.3 (b) Evaluate $\int_C 3(x^2 + y^2) ds$, where C : along the circle $x^2 + y^2 = 1$ from (1,0) to (0,1) (counter-clockwise direction) . 4

(c) Find volume of the region bounded by the first octant section cut from the region inside the cylinder $x^2 + z^2 = 1$ and by the plane $y = 0$, $z = 0$, $x = y$. 4

Que.4 (a) Find the centroid of density 1 in the plane area bounded by $y = 6x - x^2$ and $y = x$. 5

(b) Let $f(x, y) = 1$ be the density of mass in region R : $0 \leq y \leq \sqrt{1 - x^2}$; $0 \leq x \leq 1$, then find moments of inertia I_x . 3

OR

Que.4 (c) Find the centroid of density 1 in the plane area bounded by $y = 2x - x^2$ and $y = 3x^2 - 6x$. 8

Que.5 (a) State and prove Green's theorem for plane . 5

(b) Evaluate $\iint_R \overline{\nabla} \cdot \overline{V}\ dxdy$, for $\overline{V} = 7x\overline{i} - 3y\overline{j}$, C : the circle $x^2 + y^2 = 4$. 3

OR

Que.5 (c) Evaluate $\int_C [2xydx + (e^x + x^2)\, dy]$ by using Green's theorem .Also check the result by direct calculation , where C : the boundary of triangle with vertices $(0, 0), (1, 0), (1, 1)$. 8

Que.6 (a) Evaluate $\iint_S f(x, y, z)\, dA$, where $f(x, y, z) = xy$ and S : $z = xy$, $0 \leq x \leq 1$, $0 \leq y \leq 1$. 8

OR

Que.6 (b) Find moment of inertia of surface S of density 1 about z-axis ,where
$S : \bar{r} = (a + b\cos v)(\cos u\bar{i} + \sin u\bar{j}) + b\sin v\bar{k}, \quad a > b > 0 \; ; \; 0 \le u \le 2\pi \; ; \; 0 \le v \le 2\pi.$ 8

Que.7 (a) State and prove Divergence theorem of Gauss . 5

(b) By using divergence theorem , evaluate
$\iint_S [(x+z)dydz + (y+z)dzdx + (x+y)dxdy]$, where $S : x^2 + y^2 + z^2 = 4$. 3

OR

Que.7 (c) In usual notation prove that $\iiint_R \overline{\nabla}^2 f \, dv = \iint_S \frac{\partial f}{\partial n} \, dA$. 4

(d) By using divergence theorem , evaluate $\iint_S [x^2 dydz + y^2 dzdx + z^2 dxdy]$,
where S : The surface of cube $0 \le x \le 1 \; ; \; 0 \le y \le 1 \; ; \; 0 \le z \le 1$. 4

Que.8 (a) Verify Stoke's theorem for $\overline{V} = z\bar{i} + x\bar{j} + y\bar{k}$ and surface S : the square with vertices $(0,0,0), (1,0,0), (0,1,0)(1,1,0)$. 8

OR

Que.8 (b) State and prove Stoke's theorem . 8

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(3)