[43/A-9]

Time: 2 - 00 to 5 - 00 pm   M.Marks: 70

Note: (i) Simple/Scientific calculator is allowed.   (ii) Figures to the right indicate marks.
      (iii) Q.3 to 6 each sub question is of 5 marks

Q.1 Multiple Choice Questions (10×1)

(1) The joint pdf of X and Y is $f(x,y) = \frac{xy}{4}, 0 < x < 2, 0 < y < 2$ and zero otherwise, then $P(X < 1)$ is
     (a) 1/4   (b) 3/8   (c) 1   (d) 1/8

(2) If X and Y are two independent variates then
     (a) $Cov(X,Y) = 0$ (b) $f_{XY}(x,y) = f_{X(x)}f_{Y(y)}$
     (c) $f_{Y/X}(y/x) = f(y)$ (d) All of the above

(3) If $P(A) = \frac{1}{4}, P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{1}{2}$ then $P(A^c \cup B^c) =$ _____
     (a) 0.85   (b) 0.58   (c) 0.80   (d) 0.50

(4) If $V(X) = 1$ then $V(2X - 3)$ is
     (a) 5   (b) 1   (c) 4   (d) 2

(5) If $M_X(t) = (1 - 3t)^{-1}$ then $E(2 + 3X) =$ _____
     (a) 4   (b) 9   (c) 8   (d) 11

(6) The pdf of a r.v. X is then $P(X > 1/3) =$ _____
     $f(x) = \begin{cases} 1 - |x|, -1 < x < 1 \\ \ 0, otherwise \end{cases}$
     (a) 2/9   (b) 11/18   (c) 13/18   (d) None of these

(7) Which of the following is not possible in probability distribution?
     (a) $P(x) \geq 0$ (b) $\sum P(x) = 1$ (c) $\sum xP(x) = 2$ (d) $P(x) = -0.4$

(8) A r.v. X has a probability distribution as follows:

x	0	1	2	3
$P(x)$	$2k$	$3k$	$13k$	$2k$

     Where k is a positive constant, the $P(X < 2) =$ _____
     (a) 0.90   (b) 0.25   (c) 0.65   (d) 1.00

(9) The joint prob. distribution of X and Y is

X	Y	
	1	2
1	1/13	4/13
2	0	8/13

     then $P(X + Y \geq 3) =$ _____
     (a) 4/13   (b) 1/13   (c) 5/13   (d) 12/13

(10) If $M_x(t) = e^{2(e^t - 1)}$ is the m.g.f of a random variable X then $V(3 + 2X) = =$ _____
     (a) 7   (b) 2   (c) 8   (d) 4

Q.2 Short Type Questions **(Attempt Any Ten)** (10×2)

(1) State an additive law of probability for two events. Using this, Prove that $P(A') = 1 - P(A)$

(2) Find β_1 and β_2 if m.g.f. of X is $M_X(t) = e^{2(t+t^2)}$

(3) A bag contains 2 red, 3 green and 2 blue balls. Two balls are drawn at random. What is the probability that none of the balls drawn is blue?

(4) Check whether the following function is p.m.f. or not?

(1)   (P.T.O.)

$f(x) = \dfrac{1}{2x}, x = 1, 2, \ldots$ and zero otherwise

(5) The p.m.f. of a r.v. X is $P(x) = Kx, x = 1,2,3,4,5$ and zero otherwise. Determine (i) the value of K (ii) the cdf of X.

(6) Find the m.g.f of X if $f(x) = \dfrac{1}{8}(x+1), 2 < x < 4$

(7) State Bayes' theorem.

(8) Evaluate $P(A \cup B)$ if $2P(A) = P(B) = \dfrac{5}{13}$ and $P(A/B) = \dfrac{2}{5}$

(9) Verify whether $f(x,y) = 4xy, 0 \le (x,y) \le 1$ and zero otherwise, is the joint pdf of X and Y or not.

(10) The joint p.d.f of X and Y is
$f(x,y) = k(x + 3y), 0 < x < 1, 0 < y < 1$ and zero otherwise
(i) Determine the value of k (ii) the marginal distribution of X

(11) Define moment generating function. State its properties.

(12) Given that $P(X = x) = K\left(\dfrac{1}{2}\right)^x, x = 0,1,2, \ldots 6$ and zero otherwise, is the p.m.f. of r.v. X. Determine the value of K.

Q.3(a) Let A and B are two independent events with $P(A) < P(B)$. If $P(A \cap B) = \dfrac{6}{25}$ and $P(A/B) + P(B/A) = 1$ then find $P(A)$.

(b) Let A and B be two independent events. The prob. of simultaneous occurrence of these events is 1/8 and the probability of none of these occurs is 3/8, find $P(A)$ and $P(B)$.

OR

Q.3 (a) For two events A and B, Prove that
(i) $P(A \cap B) \le P(A) \le P(A \cup B) \le P(A) + P(B)$ (ii) $P(A \cap B) \ge P(A) + P(B) - 1$

(b) Probability of solving specific problem independently by A and B are $\dfrac{1}{2}$ and $\dfrac{1}{3}$ respectively. If try to solve the problem independently, find the probability that (i) the problem is solved (ii) exactly one of them solves the problem

Q.4(a) An urn contains 5 white and 2 black marbles. If 3 marbles are to be selected at random and let X denote the no. of black marbles, find the probability distribution and c.d.f of X. Determine the prob. that a randomly selected marbles have atleast one black marble.

(b) If $f(x) = \begin{cases} kx, & 0 \le x < 3 \\ k(6 - x), & 3 \le x < 6 \\ 0, & x \ge 6 \end{cases}$
is the pdf of X then find (i) k (ii) the c.d.f of X (iii) $P\left(\dfrac{1}{2} < X < 4\right)$

OR

Q.4 (a) The probability distribution of a r.v. X is given below:

x	1	2	3	4	5	6	7	8
$P(X = x)$	k	k	$3k$	$2k$	$k^2 + k$	$2k^2$	$4k^2 + k$	$3k^2$

Find (i) k (ii) the c.d.f. of X (iii) the minimum value of k so that $P(X \le k) = \dfrac{1}{2}$

(b) Given that $f(x) = k\left(\dfrac{1}{2}\right)^x, x = 0,1,2, \ldots 6$ and zero otherwise, is the probability distribution of X. Find k and c.d.f of X.

Q.5(a) The probability distribution of X is
If $f(x) = \dfrac{1}{10}, -5 < x < 5$ and zero otherwise. Find the m.g.f of X and hence mean and variance of X.

(b) The moment generating function of X is $M_X(t) = (3 - 2e^t)^{-1}$, Find β_1 and β_2

OR

Q.5 (a) If $f(x) = 5e^{-5x}, x > 0$ and zero otherwise, is the p.d.f. of X. Find the m.g.f, c.g.f and β_1 and β_2.

(b) From a group of 5 men and 3 women, a committee of 3 members is selected. If X represents the no. of

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women in the committee, find $V(X)$.

Q.6 (a) Let X and Y be two continuous r.v's with the joint probability density function

$f(x,y) = 24xy, 0 < x < 1, 0 < y < 1, x + y < 1$ *and zero otherwise*

Find (i) the marginal distribution of Y (ii) Calculate $P(X > 1/3)$

(b) Consider two r.v's X and Y with joint probability distribution given in the following table:

X	Y		
	2	4	5
1	1/12	1/24	1/24
2	1/6	1/12	1/8
3	1/4	1/8	1/4

Find (i) $P(X \leq 2, Y \leq 4)$ (ii) $P(Y = 2/X = 1)$ (iii) the marginal distribution of X and Y (iv) Are X and Y independent?

OR

Q.6 The joint pdf of X and Y is $f(x,y) = cxy, 0 < x < 4, 1 < y < 5$ *and zero otherwise*

Determine (i) the constant c (ii) $P(X \geq 3, Y \leq 2)$ (iii) $P(X + Y < 3)$

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