[46/A-15] **Seat No :**________ No of printed pages : 3

SARDAR PATEL UNIVERSITY

B.Sc.(SEMESTER - III) EXAMINATION (NC)

Friday , 29^{th} Nov.,2019

US03EMTH05 (CALCULUS AND ALGEBRA - I)

Time : 2:00 p.m. to 4:00 p.m. **Maximum Marks : 70**

Que.1 Attempt the following. 10

(1) $\lim_{x\to 0}\frac{\sin x}{x} = \ldots\ldots$

(a) 0 (b) x (c) 1 (d) none

(2) $\lim_{x\to 0}\frac{\tan 2x}{x} = \ldots\ldots$

(a) 0 (b) 2 (c) 1 (d) 2x

(3) $\lim_{x\to 0}\frac{2x}{\tan 2x} = \ldots\ldots$

(a) 2 (b) 1 (c) 1/2 (d) 0

(4) If $u = e^y \cos x$ then $u_y = \ldots\ldots$

(a) $-e^y \sin x$ (b) $-e^y \cos x$ (c) $e^y \sin x$ (d) $e^y \cos x$

(5) If $u = e^x \cos y$ then $\frac{\partial u}{\partial x} = \ldots\ldots$

(a) $-e^x \cos y$ (b) $e^x \cos y$ (c) $e^x \sin y$ (d) $-e^x \sin y$

(6) If $u = e^x \cos y$ then $u_{xy} = \ldots\ldots$

(a) $-e^x \cos y$ (b) $e^x \cos y$ (c) $e^x \sin y$ (d) $-e^x \sin y$

(7) $\overline{(\overline{A})} = \ldots\ldots$

(a) A (b) A^θ (c) $\overline{A}$ (d) A'

(8) If $A = \begin{bmatrix} 1 & 3 \\ 4 & 6 \\ 0 & 4 \end{bmatrix}$ then order of A is ....

(a) 2x3 (b) 3x3 (c) 2x2 (d) 3x2

(9) Reversal law for the transpose of product is.....

(a) $(AB)' = B'A'$ (b) $(AB)' = A'B'$ (c) $(AB)' = BA$ (d) $(AB) = BA$

(10) Associative law for matrix multiplication is .....

(a)$(AB)C = A(CB)$ (b) $A(BC) = (AB)C$ (c) $A + B = B + A$ (d) $AB = BA$

(P.T.O)

Que.2 Attempt the following.(**Any Ten**) 20

(1) Evaluate $\lim_{x\to\infty} \left[\frac{1^{\frac{1}{x}}+2^{\frac{1}{x}}+3^{\frac{1}{x}}}{3}\right]^{3x}$.

(2) Evaluate $\lim_{x\to 2} \frac{\tan(x^2-4)}{x-2}$.

(3) Evaluate $\lim_{x\to a} (a-x)\tan\left(\frac{5\pi x}{2a}\right)$.

(4) For $u = x^3+3x^2y$ find $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}$.

(5) For $u = y^3+3x^2y^2$ prove that $\frac{\partial^2 u}{\partial x\partial y} = \frac{\partial^2 u}{\partial y\partial x}$

(6) For $u = x^3+4xy^2$ find $\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial y^2}$.

(7) Define Transpose of matrix,Conjugate of matrix with example.

(8) Define Conjugate transpose of matrix,Triangular matrix with example.

(9) Define Skew symmetric matrix,Hermitian matrix with example.

(10) If $A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$ then find A^2-3A .

(11) If $A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$ then find A^3 .

(12) If $A = \begin{pmatrix} 13 & 16 & 19 \\ 14 & 17 & 20 \\ 15 & 18 & 21 \end{pmatrix}$ then find $|A|$.

Que.3 (a) Evaluate $\lim_{x\to 0} \frac{e^x+\log(1-x)-1}{\tan x - x}$ 5

(b) Evaluate $\lim_{x\to 0} \left(\frac{1}{2x^2}-\frac{\cot^2 x}{2}\right)$ 5

OR

Que.3 (c) Evaluate $\lim_{x\to 0} \frac{\log(\log(1-3x^2))}{\log(\log(\cos 2x))}$ 5

(d) Evaluate $\lim_{x\to 1} (4-4x^2)^{\frac{1}{\log(2-2x)}}$. 5

Que.4 (a) State and prove Euler's theorem for function of two variables . 5

(b) If $u = \sin^{-1}\left(\frac{x^2y^2}{x+y}\right)$ then prove the following.

(i) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y} = 3\tan u$

(ii) $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2} = 3\tan u(3\sec^2 u-1)$ 5

OR

Que.4 (c) Verify Euler's theorem for $z = x^n \log \frac{y}{x}$ .Also find $x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2}$ 5

(d) If $z = f(x,y)$, $x = r\cos\theta, y = r\sin\theta$ then prove that

$$\left[\frac{\partial z}{\partial x}\right]^2 + \left[\frac{\partial z}{\partial y}\right]^2 = \left[\frac{\partial z}{\partial r}\right]^2 + \frac{1}{r^2}\left[\frac{\partial z}{\partial \theta}\right]^2 .$$ 5

Que.5 (a) Prove that Every square matrix can be expressed in one and only one way as the sum of a symmetric and skew symmetric matrix. 5

(b) For $A = \begin{pmatrix} 0 & 2m & n \\ l & m & -n \\ l & -m & n \end{pmatrix}$, show that $AA' = I$; where $l = \frac{1}{\sqrt{2}}, m = \frac{1}{\sqrt{6}}, n = \frac{1}{\sqrt{3}}$. 5

OR

Que.5 (c) Prove that Every square matrix can be expressed in one and only one way as $P + iQ$, where P and Q are Hermitian matrices. 5

(d) If $A = \begin{pmatrix} 1+i & 3-i & 3+5i \\ 1-3i & 2i & -6i \\ 3+i & 1-5i & -2+-i \end{pmatrix}$ then find $(\overline{A})'$ and AA^θ . 5

Que.6 (a) State and prove Cayley-Hamilton theorem. 5

(b) For $A = \begin{pmatrix} 2 & 2 \\ 3 & -4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 7 \\ 5 & 3 \end{pmatrix}$, $C = \begin{pmatrix} -1 & 5 \\ 3 & 2 \end{pmatrix}$, prove that $(A+C)B = AB+CB$ 5

OR

Que.6 (c) If $A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$ then show that $A^k = \begin{pmatrix} 1+2k & -4k \\ k & 1-2k \end{pmatrix}$ where k is any positive number. 5

(d) If $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ then find out the values of α, β such that $(\alpha I + \beta A)^2 = A$. 5

—x—