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SARDAR PATEL UNIVERSITY
B.Sc.(SEMESTER - III) EXAMINATION (NC)
Thursday, 28^{th} November , 2019
US03EMTH01 (CALCULUS)

Time : 02.00 p.m. to 04.00 p.m. Maximum Marks:70

Que.1 Attempt the following. 10

(1) $\int_0^{\pi/2} \sin x dx$ is ....

(a) Proper Integral(b) Improper integral of 1st kind (c) Improper integral of 2nd kind (d) none

(2) $\int_1^{\infty} x dx$ is ....

(a) Proper Integral (b) Improper integral of 1st kind (c) Improper integral of 2nd kind (d) none

(3) For $n \geq 0$; Γn=..........

(a) $n!$ (b) n (c) $(n-1)!$ (d) $(n-1)$

(4) $\beta(p,q) = \beta(p+1,q)$ .... $\beta(p,q+1)$

(a) + (b) - (c) · (d) /

(5) If $f(x,y,z) = x^3 + 2y + 6z$ then $\overline{\nabla} f$=.......

(a) $3x^2\overline{i} + 2\overline{j} + 6\overline{k}$ (b) $x^2\overline{i} + 2\overline{j} + 6\overline{k}$ (c) $x^3\overline{i} + 2\overline{j} + 6\overline{k}$ (d) $3x^2\overline{i} + 2y\overline{j} + 6\overline{k}$

(6) If $\overline{v} = 2x^2\overline{i} + 3y^2\overline{j}$ then $\overline{\nabla} \times \overline{v}$ =......

(a) 1 (b) $\overline{0}$ (c) -1 (d) None

(7) If $\overline{v} = 2x^2\overline{i} + 3y^2\overline{j}$ and $f = 4x + 3y$ then $\overline{v} \times (\overline{\nabla} f)$ =......

(a) $(6x^2 - 12y^2)\overline{k}$ (b) $6x^2\overline{i} + 12y^2\overline{j}$ (c) $x^2\overline{i} - 12y^2\overline{j}$ (d) $6x^2\overline{i} - y^2\overline{j}$

(8) $f(x)$ is odd function if......

(a) $f(-x) = -f(x)$ (b) $f(-x) = f(x)$ (c) $f(-x) = -f(-x)$ (d) $f(x) = -x$

(9) Graph of odd function has symmetry with respect to ....

(a) X - axis (b) Y- axis (c) Origin (d) None

(10) Each term of trigonometric series has the period ......

(a) π (b) 2π (c) 3π (d) $\frac{\pi}{2}$

(P.T.O)

Que.2 Attempt the following (Any Ten) 20

(1) Evaluate $\int_{4}^{\infty} \frac{dx}{\sqrt{x^3}}$.

(2) Evaluate $\int_{0}^{1} \frac{dx}{\sqrt{1-x^2}}$.

(3) Evaluate $\int_{0}^{1} \frac{dx}{\sqrt[3]{x}}$.

(4) Prove that $\Gamma n = (n-1)!$.

(5) Prove that $\left|\frac{1}{2}\right. = \sqrt{\pi}$.

(6) Evaluate $\int_{0}^{\infty} e^{-x^2} dx$.

(7) Prove that $\overline{\nabla}(f \pm g) = \overline{\nabla} f \pm \overline{\nabla} g$.

(8) Prove that $\overline{\nabla}(fg) = f\overline{\nabla} g + g\overline{\nabla} f$.

(9) Prove that $\overline{\nabla}\left(\frac{f}{g}\right) = \frac{g\overline{\nabla} f - f\overline{\nabla} g}{g^2}$.

(10) Define Periodic function and Period of function .

(11) Define Trigonometric series , Fourier series .

(12) Define Even and Odd function , Half range expansions .

Que.3 (a) Prove that the integral $\int_{a}^{\infty} \frac{dx}{x^{\mu}}$ $(a > 0)$ is convergent if and only if $\mu > 1$. 5

(b) Examine convergence of $\int_{1}^{\infty} \frac{x\,dx}{(1+x)^3}$. 5

OR

Que.3 (c) Prove that the integral $\int_{a}^{b} \frac{dx}{(x-a)^{\mu}}$ is convergent if and only if $\mu < 1$. 5

(d) Examine convergence of $\int_{1}^{\infty} \frac{dx}{x^{1/3}(1+x)^{1/2}}$. 5

Que.4 (a) Prove that $\beta(m,n) = \int_{0}^{1} \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}\, dx$. 5

(b) Prove that $\int_{0}^{\infty} \frac{x^{m-1}dx}{(a+bx)^{m+n}} = \frac{1}{a^n b^m}\beta(m,n)$. 5

OR

Que.4 (c) Evaluate $\int_{0}^{\infty} \frac{x^9 dx}{(2+4x)^{12}}$ 5

(d) Prove that $\frac{\beta(p,q+1)}{q} = \frac{\beta(p+1,q)}{p} = \frac{\beta(p,q)}{p+q}$. 5

Que.5 (a) Prove that $\overline{\nabla} \cdot (f\overline{\nabla}g + g\overline{\nabla}f) = f\overline{\nabla}^2 g + g\overline{\nabla}^2 f + 2\overline{\nabla}f \cdot \overline{\nabla}g$. 5

(b) Find directional derivative of $f(x,y,z) = 2x^2 + 3y^2 + z^2$ at point (2,1,3) in the direction of $\bar{a} = \bar{i} - 2\bar{k}$. 5

OR

Que.5 (c) Prove that $\overline{\nabla}^2(fg) = f\overline{\nabla}^2 g + g\overline{\nabla}^2 f + 2\overline{\nabla}f \cdot \overline{\nabla}g$. 5

(d) Prove that $\overline{\nabla} \cdot (\bar{u} \times \bar{v}) = \bar{v} \cdot (\overline{\nabla} \times \bar{u}) - \bar{u} \cdot (\overline{\nabla} \times \bar{v})$. 5

Que.6 (a) Find Euler's constant a_n, b_n for Fourier series of a function $f(x)$ over $[-\pi, \pi]$. 6

(b) Represent the function $f(t) = 1 \ , \ 0 < t < l$ by Fourier cosine series . 4

OR

Que.6 (c) Find the Fourier coefficient of periodic function $f(x) = \begin{cases} -k & \text{if } -\pi < x \leq 0 \\ k & \text{if } 0 < x < \pi. \end{cases}$ 6

(d) Find Fourier coefficient of periodic function $f(x) = x^2 \ , \ -\pi < x < \pi$. 4

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