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Sardar Patel University

B.Sc. Semester-III Examination

26th November, 2019

Tuesday

Subject:- Elements of Probability Theory

Paper Code:-US03CSTA22

Time:- (02:00 P.M. to 5:00 P.M.) **M.Marks:70**

Note:- Simple/ Scientific calculator is allowed.

Q.1. Multiple Choice Questions:- [10]

(1) The probability of drawing one white ball from a bag containing 6 red, 8 black,10 yellow and 1 green balls is_____.

(a)$\frac{1}{25}$ (b) 0 (c)$\frac{24}{25}$ (d)$\frac{15}{20}$

(2) A and B are two independent events such that $P(\bar{A})$ = 0.7, $P(\bar{B})$=k and P(AUB)=0.8, then k is_____.

(a)$\frac{5}{7}$ (b) 1 (c)$\frac{2}{7}$ (d)noneofthese

(3) In the simultaneously tossing of two perfect coins, the probability of obtaining 4 as the sum of the resultant faces is_____.

(a)$\frac{4}{12}$ (b)$\frac{3}{12}$ (c)$\frac{1}{12}$ (d)noneofthese

(4) Let the distribution function of a random variable X be

$F(x)=\begin{cases}1-e^{-2x}, & x \geq 0\\ 0, otherwise\end{cases}$ .Then the density function is_____.

(a) $\begin{cases}1-e^{-2x}, & x>0\\ 0, otherwise\end{cases}$ (b) $\begin{cases}1-2e^{-2x}, & x>\\ 0, otherwise\end{cases}$ (c) $\begin{cases}2e^{-2x}, & x>0\\ 0, otherwise\end{cases}$ (d)noneofthese

(5) Var(c) =_____, where c is a constant.

(a)Var(x) (b) 0 (c) 1 (d)noneofthese

(6) If $E(X) = \int_{-\infty}^{\infty} x.f(x)dx$. then the random variable X is of the _____ random variable.

(a) continuous (b) Discrete (c)both(a)&(b) (d)noneofthese

(7) If $M_X(t)$ is the m.g.f. of a random variable X and Y = aX + b then $M_Y(t)$ = ___

(a) $e^{bt}M_X(at)$ (b) $e^{at}M_X(bt)$ (c) $M_X(at+b)$ (d) none of these

(8) If for two random variables X and Y, E(X) = 5 , E(Y) = 4, V(X) = 25 , V(Y) = 16 and E(X.Y) = 5 then r = _____.

(a) 0.75 (b) - 0.75 (c) - 1 (d)noneofthese

(9) If f(x,y) = 4xy , $0 < x < 1$; $0 < y < 1$,
= 0 , elsewhere is the joint p.d.f. of X & Y then
P(X < 0.50 , Y < 0.50) = _____.

(a) $\frac{1}{16}$ (b) $\frac{1}{32}$ (c) $\frac{1}{64}$ (d) none.

(10) If f(x,y) is the joint p.d.f. of two independent random variables X and Y, then f(x,y) = _____.

(a) f1(x). f2(y) (b) $f_1(x)/f_2(y)$ (c) $f_1(x) + f_2(y)$ (d)noneofthese

Q.2. Short Type Questions:- (Attempt Any Ten) [20]

(1) Prove in usual notations for three events A, B and C of sample space S, that P(AUB /C) = P(A /C) + P(B /C) - P(A∩B /C).

(2) A and B are mutually exclusive events.
(i)Given P (A) = $\frac{3}{5}$ and P (B) = $\frac{1}{5}$ .Find P(A or B).

(ii)Given P (A') = 0.65 & P (AU B) = 0.65 .Find P(B).

(3) Write an appropriate sample space for tossing of two fair dice. Find the probability for the following events. (i) the sum is greater than 10. (ii) the first die shows an even number. (iii) an even number on first die or a sum of 8 in single throw of two die.

(4) Consider the experiment of tossing of three fair coins. Let variable X denote the number heads. Find the probability mass function and cumulative distribution function.

(5) The table below shows the prob. of X

x	1	2	3	4	5
p(x)	2k	3k	4k	5k	6k

(i)For what value of k is the prob. is p.m.f or probability function.
(ii)Find P ($3 \leq X \leq 5$)

(6) If $f(x) = kx^2$, $0 < x < 1$
= 0, otherwise, Find k and P($1/3 < X < 2/3$).

(7) If $M_X(t) = e^{5(e^t - 1)}$ is the m.g.f. of a random variable X then show that all the cummulants are same.

(8) If $P(t) = \left(\frac{1}{3} + \frac{2}{3}t\right)^4$ then find the p.m.f. of x.

(9) The following table shows the pmf of a r.v. x

x	0	1	2	k
f(x)	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$

Find k if mean = 1.5. Also, find V(X).

(10) Define joint p.d.f. of two r.vs. X and Y. Define marginal values of X & Y.

(11) Is $f(x,y) = \frac{(2x+3y)}{120}$, x=1,2,3; y=1,2
= o, otherwise, the joint p.m.f. of X and Y?

(12) If f(x,y) = k(x + y) , x = 1,2,3. ; y = 1, 2, 3.
= 0, otherwise. Find k and P(X + Y <3).

Q.3. (a) Three coins are tossed. Find the prob. of getting [05]
(i)all heads (ii) at least two heads (iii) at most 2 heads (iv) no head (v) exactly two tails (vi) a head on first coin.

(b) Two students X and Y appeared in examination. The prob. that X will qualify the examination is 0.05 and that Y qualify the examination is 0.10. The prob. that both will qualify the examination is 0.02. Find the pob. that (i) both X and Y will not qualify the exam (ii) at least one of them will qualify the exam (iii) only one of them will qualify the exam (iv) at least one of them will not qualify. [05]

OR

Q.3. (a) Two cards are drawn from a pack of 52 cards. What is the prob. that either both are red or both are king? [04]

(b) In 2002 there will be three candidates for the position of principal- Mr.X, Mr.Y and Mr.Z- whose chance of getting the appointment are in proportion 4:2:3 respectively. The probability that Mr. X if selected would introduce co-education in the college is 0.3. The probability of Mr. Y and Mr. Z doing the same are respectively 0.5 and 0.8. [06]
(i)What is probability that there will be co-education in the college in 2003.
(ii)If there is co-education in the college in 2003, what is the probability

that Mr. Z is the principal?

Q.4. (a) A discrete r.v. has the p.m.f. [05]

x	1	2	3	4	5	6	7
p(x)	k	2k	2k	3k	$2k^2 + k$	$3k^2$	$5k^2$

Determine k (ii) P ($X \leq 4$) (iii) P ($2 < X < 7$) (iv) Find the min. value of a such that P ($X \leq a$) < 0.5.

(b) If f(x) = ax, $0 < x < 3$ [05]
= a(6-x) , $3 \leq x < 6$
= 0, otherwise is the p.d.f. of x then find (i) a (ii) P($2 < x < 4$)
(iii) P ($x < 5$) (iv)P($x > 2$). (v)cdf.

OR

Q.4. (a) A continuous r.v. X has p.d.f. given by [05]
f(x) = c (x+3), $2 < x < 8$
= 0, otherwise
(i)Determine the value of c.(ii)P($3 < x < 5$)(iii)P($x \geq 4$)(iv)c.d.f. of a r.v. x.

(b) If f(x) = 2 x , $0 < x < 1$ [05]
= 0, otherwise is the p.d.f. of x then find c.d.f. of x and hence the Median.

Q.5. (a) If $f(x) = \frac{x}{21}$, x = 1,2,3,.....,6. [04]
= 0, otherwise, is the pmf of a discrete r.v. X then find E(X),V(X), E(2 x +3).

(b) If $f(x) = 5\, e^{-5x}$, $0 < x < \infty$ [06]
= 0, otherwise is the pdf of a rv X then find mgf, cgf and β_1 , β_2

OR

Q.5. (a) If $f(x) = (\frac{2}{3})(\frac{1}{3})^x$, x = 0,1,2,.... [06]
= 0 , otherwise is the pm of a discrete r.v. X, obtain r^{th} factorial moment. And hence or otherwise its mean and variance.

(b) If $f(x) = (\frac{1}{2})^{x+1}$, x= 0,1,2,..... [04]
= 0, otherwise, is the pmf of a discrete r.v. X then show that variance = 2 Mean.

Q.6. (a) If f(x,y) = k (3 x + 2 y) , x = 1,2,3,4; y=1,2,3 [05]
= 0 , otherwise is the joint p.m.f. of X and Y.
Find (i) k (ii) the marginal distribution of X and Y (iii) P($X < 1, Y < 1$)
(iv)P($x+y<4$)

(b) If f(x,y) = k xy , $0<x<2$, $0<y<2$ [05]
= 0 , otherwise is the joint p.d.f. of x and y then find (i)k
(ii) P($1<x<2$) (iii) P($1 < y < 1.5$) (iv) P($0<x<1$, $1<y<2$).

OR

Q.6. For the bi variate prob. distribution of X and Y find [10]
(i)P ($X \leq 1, Y = 2$) (ii) P ($X \leq 1$) (iii) P ($Y \leq 3$) (iv) P ($X < 3, Y \leq 4$)
(v) the marginal distribution of X and Y (vi) conditional distribution of X given Y =1 (vii) conditional distribution of Y given X =1.

X\ Y	1	2	3	4	5	6
0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{2}{64}$	$\frac{2}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$