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SARDAR PATEL UNIVERSITY

B.Sc.(SEMESTER III) EXAMINATION

Saturday , 30^{th} Nov., 2019

MATHEMATICS : US03CMTH22

(MULTIVARIATE CALCULUS)

Time : 02.00 p.m. to 05.00 p.m. Maximum Marks:70

Que.1 Fill in the blanks. 10

(1) The value of $\int_0^\infty (1+2x)e^{-x}\,dx =$ ....

(a) 0 (b) 1 (c) 3 (d) ∞

(2) The value of $\int_0^3 \frac{1}{\sqrt{9-x^2}}\,dx =$ ....

(a) 0 (b) 1 (c) $\pi/2$ (d) ∞

(3) For the curve $x^2+y^2=1$, $\frac{ds}{dt} =$ ....

(a) 0 (b) 1 (c) $\sqrt{2}$ (d) -1

(4) In double integral , Total mass M of density 1 over region $0 \le x, y \le 2$ is ....

(a) 1 (b) 2 (c) 0 (d) 4

(5) $\int_0^2\int_0^y dxdy =$ ....

(a) 1 (b) 1/2 (c) 0 (d) 2

(6) $xdx+ydy+zdz =$ ....

(a) $d\left[\frac{x^2+y^2+z^2}{2}\right]$ (b) $d\left[\frac{x^2+y^2+z^2}{3}\right]$ (c) $d\left[x^2+y^2+z^2\right]$ (d) $d\left[\frac{(x+y+z)^2}{2}\right]$

(7) Area of plane region $r = a(1+\cos\theta)$ is A = ...

(a) $\frac{3\pi a^2}{2}$ (b) $\frac{3\pi a}{2}$ (c) $\frac{\pi a^2}{2}$ (d) $\frac{3a^2}{2}$

(8) If $f = -xy^2$, $g = x^2y$ then $\frac{\partial f}{\partial y} - \frac{\partial g}{\partial x} =$ ....

(a) $4xy$ (b) $-4xy$ (c) $2xy$ (d) $-2xy$

(9) $\int_0^1\int_0^2\int_0^3 dxdydz =$ ....

(a) 1 (b) 6 (c) 3 (d) 2

(10) If $\bar{n} = \bar{j}$ then $dA =$ ....

(a) 0 (b) $dxdz$ (c) $dxdy$ (d) $dydz$

(P.T.O.)

Que.2 Answer the following (Any Ten) 20

(1) Prove that div $(f\mathrm{U}) = f(\mathrm{div}\,\mathrm{U}) + (\mathrm{grad}\, f)\cdot \mathrm{U}$.

(2) Show that curl $(r^n \mathrm{r}) = 0$, where $\mathrm{r} = x\mathrm{i} + y\mathrm{j} + z\mathrm{k}$ and $r = |\mathrm{r}|$.

(3) Show that $\overline{|m} = 2\int_0^\infty e^{-x^2} x^{2m-1}\, dx$.

(4) Evaluate $\int_C 3(x^2+y^2)\, ds$, where C : Over the path $y = x$ from (0,0) to (1,1) (counterclockwise direction) .

(5) Evaluate $\int_0^1 \int_x^{2x} (1+x^2+y^2)\, dydx$.

(6) Change the order of integration in $\int_0^2 \int_0^x f(x,y)\, dydx$.

(7) Obtain first fundamental form of $\overline{r} = a\cos v\cos u\overline{i} + a\cos v\sin u\overline{j} + a\sin v\overline{k}$.

(8) Find equation of tangent plane and normal line to the surface $y^2 + x^2 = z$ at $(2,1,5)$.

(9) Evaluate $\int_C \frac{\partial w}{\partial n}\, ds$ for $w = 2x^2 + y^2$ and C : the boundary of the region bounded by $y = x^2$ and $y = x + 2$.

(10) By using divergence theorem , evaluate $\iint_S [(x+z)dydz + (y+z)dzdx + (x+y)dxdy]$, where S: $x^2+y^2+z^2 = 4$.

(11) Let R be a closed region in space and S be its boundary , let g be harmonic function in R , if $\frac{\partial g}{\partial n} = 0$ on S then prove that g is constant function in R .

(12) Evaluate $\int_0^1 \int_0^x \int_0^{1-x^2} z\, dydzdx$.

Que.3 (a) State and prove Legendre's Formula . 5

(b) Prove that $\mathrm{grad}(\mathrm{U}\cdot\mathrm{V}) = \mathrm{U}\times \mathrm{curl}\,\mathrm{V} + \mathrm{V}\times\mathrm{curl}\,\mathrm{U} + (\mathrm{V}\cdot\nabla)\mathrm{U} + (\mathrm{U}\cdot\nabla)\mathrm{V}$. 5

OR

Que.3 (c) Prove that $\beta(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\, dx$ .Hence prove that $\beta(m,n) = \int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}\, dx$. 6

(d) Find a scalar function f such that $\mathrm{V} = \nabla f$, where $\mathrm{V} = e^{xyz}(yz\mathrm{i} + xz\mathrm{j} + xy\mathrm{k})$. 4

Que.4 (a) Transform $\iint_R (x^2+y^2)\, dxdy$ in uv-plane by taking $x+y = u, x-y = v$.Then evaluate it, where R:Parallelogram with vertices (0,0),(1,1),(2,0),(1,-1). 5

(b) Find the centroid of density 1 in the plane area bounded by $y = 2x - x^2$ and $y = 3x^2 - 6x$. 5

OR

Que.4 (c) Find volume of the region bounded by the cylinder $x^2+y^2 = 1$, $y^2+z^2 = 1$. 5

(d) Change the order of integration in $\int_0^a \int_{\sqrt{a^2-x^2}}^{x+2a} f(x,y)\, dydx$. 5

Que.5 (a) Verify the result $\iint_R \nabla \cdot \overline{V}\, dxdy = \int_C \overline{V} \cdot \overline{n}\, ds$ for $\overline{V} = 7x\,\overline{i} - 3y\,\overline{j}$ $C:$ the circle $x^2 + y^2 = 4$. 5

(b) Find area of the surface $z^2 = x^2 + y^2$, where $0 \leq z \leq 1$. 5

OR

Que.5 (c) State and prove Green's theorem for plane . 5

(d) Find moment of inertia of surface S of density 1 about z-axis , where $S : \overline{r} = (a + b\cos v)(\cos u\,\overline{i} + \sin u\,\overline{j}) + b\sin v\,\overline{k},\ \ a > b > 0\ ,\ 0 \leq u \leq 2\pi\ ,\ \ 0 \leq v \leq 2\pi.$ 5

Que.6 (a) State and prove divergence theorem of Gauss . 5

(b) Verify Stoke's theorem for $\overline{V} = 3y\,\overline{i} - xz\,\overline{j} + yz^2\,\overline{k}$ and surface S : $2z = x^2 + y^2$ bounded by $z = 2$. 5

OR

Que.6 (c) By using triple integral ,find volume of the region R : in the first octant bounded by $x^2+z^2 = 1$ and by the plane $y = 0, z = 0, x = y$. 5

(d) Verify Stoke's theorem for $\overline{V} = (x^2 + y^2)\,\overline{i} - 2xy\,\overline{j}$ and surface S : the rectangle bounded by the lines $x = \pm a$, $y = 0$, $y = b$. 5

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