Seat No : ______ No. of Printed Pages : 03

[27]

SARDAR PATEL UNIVERSITY
B.SC.SEM-III EXAMINATION
05^{th} December 2019, Thursday
02.00 p.m. to 05.00 p.m.
US03CMTH02
(Numerical Analysis)

Maximum Marks: 70

Q.1 Choose the correct option in the following questions, mention the correct option in the answerbook. [10]

(1) In iteration method the sequence of approximation converges to the root provided
(a) $|\phi'(x)| > 1$ (b) $|\phi'(x)| < 1$ (c) $|\phi(x)| \neq 1$ (d) $|\phi(x)| < 1$

(2) If $y_5 = 4$ and $y_{15} = 10$ then $E^5 y_{10} =$
(a) 10 (b) 5 (c) 15 (d) 20

(3) The interval $[-2, 2]$ is divided into 8 equal subintervals then the length of each subinterval is ......
(a) 2 (b) 0.8 (c) -2 (d) 0.5

(4) The algebraic sum of any difference column in the difference table is
(a) 0 (b) 1 (c) any number (d) positive number

(5) By putting $n =$ .... in the General formula for integration, we get simpson's $\frac{1}{3}$ rule.
(a) 1 (b) 2 (c) 3 (d) -1

(6) For a set of tabulated points $(x_0, y_0), (x_1, y_1), \dots (x_n, y_n)$ with equal spaced arguments, the interpolation for x near x_n is best given by...
(a) Gauss's froward formula (b) Newton's froward formula
(c) Gauss's backward formula (d) Newton's backward formula

(7) Trapezoidal rule is obtain from.....
(a) Simpson's $\frac{1}{3}$ rule (b)General formula for integration
(c) Simpson's $\frac{3}{8}$ rule (d)Gauss formula

(8) The Lagrange's interpolation formula is applicable for ... arguments.
(a) Unequal spaced (b) Equal spaced (c) positive (d) negative

(9) In forward differences $\Delta^2 y_0 =$ ....
(a)$\Delta y_0 - \Delta y_1$ (b)$y_2 - 2y_1 + y_0$ (c)$y_2 + 2y_1 + y_0$ (d)$\Delta y_2 - \Delta y_0$

(10) For a function $f(x)$ if $f(a) < 0$ and $f(b) > 0$ then, there exist:......... $x \in (a, b)$ such that $f(x) = 0$.
(a) Exactly one (b) at least one (c) at most one (d) none of them

Q.2 Attempt **any ten** in short: [20]

(1) Discuss the method of successive approximations in short.

(2) Derive a formula to calculate the cube root of a number N.

(P.T.O.)

(3) Prove that $E^{-1} = 1 - \nabla$.

(4) Explain interpolation.

(5) If $\phi(x) = (2x+5)^{(1/3)}$ then find $|\phi'(x)|$ for x=2.5 .

(6) In usual notations show that $\Delta - \nabla = \Delta\nabla$.

(7) State Newton's forward interpolation formula for equally spaced arguments.

(8) Define averaging and shift operators.

(9) State Romberg Integration formula.

(10) Define divided differences.

(11) State Trapezoidal rule.

(12) State Simpson's $\frac{1}{3}$ rule.

Q.3(a) Obtain the real root of the equation $x^3 - 2x - 5 = 0$ correct upto three decimal points using false position method. [6]

(b) Explain Aitken's Δ^2procedure of approximation to find real of equation. [4]

OR

Q.3(c) Obtain the real root of the equation $x^3 + x - 4 = 0$ correct upto three decimal points using bisection method. [6]

(d) Using the formula $x_{n+1} = \frac{1}{2}[x_n + \frac{N}{x_n}]$, calculate the square root of 7. [4]

Q.4(a) The populations of a town were as under: [6]

Year	1891	1901	1911	1921	1931
Populations (in thousands)	46	66	81	93	101

Estimate the population for the year 1895 and 1925.

(b) In usual, Prove that (i) $\Delta - \nabla = \delta^2$, (ii) $\mu = \sqrt{1 + \frac{1}{4}\delta^2}$. [4]

OR

Q.4(c) Derive Newton's forward interpolation formula for equally spaced values of argument. [6]

(d) Locate and correct the error in the following table of values [4]

x	2.5	3.0	3.5	4.0	4.5	5.0	5.5
y	4.32	4.83	5.27	5.47	6.26	6.79	7.23

Q.5(a) Obtain first and second order numerical differentiation formula for Newton's backward difference formula. [5]

(b) Using Lagrange's interpolation formula express the function $\frac{x^2+6x+1}{(x-1)(x+1)(x-4)(x-6)}$ as sums of partial fraction. [5]

OR

Q.5(c) Derive Newton's divided difference formula. [5]

(d) Show that the divided differences are symmetrical in their arguments. [5]

Q.6(a) Derive the general formula for numerical integration. [5]

(b) Evaluate $\int_0^1 \frac{1}{1+x} dx$, correct to three decimal places using Simpson's $\frac{1}{3}$ rule and by taking $h = 0.05$. [5]

OR

Q.6(c) Derive the formula of Simpson's $\frac{3}{8}$ rule. [5]

(d) Use Picard's method to approximate y when $x = 0.25$ and $x = 0.5$, given that $y(0) = 0$ and $\frac{dy}{dx} = \frac{x^2}{y^2+1}$ correct to three places. [5]

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