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SARDAR PATEL UNIVERSITY
B.Sc.(SEMESTER III) EXAMINATION (NC)
Wednesday , 27^{th} Nov., 2019
MATHEMATICS : US03CMTH01
(ADVANCED CALCULUS)

Time : 02.00 p.m. to 05.00 p.m. Maximum Marks:70

Que.1 Fill in the blanks. 10

(1) $\frac{ds}{dt}$ = ...

(a) $\left|\frac{d\bar{r}}{dt}\right|$ (b) $\frac{d\bar{r}}{dt}$ (c) $\left|\frac{d\bar{r}}{ds}\right|$ (d) $\left|\frac{dr}{dt}\right|$

(2) For the curve $x^2 + y^2 = 1$, $\frac{ds}{dt}$ = ...

(a) 0 (b) 1 (c) $\sqrt{2}$ (d) -1

(3) For the curve $y = -x$, $\frac{ds}{dt}$ = ...

(a) 2 (b) 1 (c) $\sqrt{2}$ (d) $-\sqrt{2}$

(4) For $x + y = u$, $x - y = v$, jacobian J = ...

(a) -2 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{-1}{2}$

(5) $yzdx + xzdy + xydz$ = ...

(a) $d\left[\frac{x^2+y^2+z^2}{2}\right]$ (b) $d[xyz]$ (c) $d[x^2+y^2+z^2]$ (d) $d\left[\frac{xyz}{2}\right]$

(6) Area of limacon $r = a(1 + \cos\theta)$ in the first and second quadrant is A = ...

(a) $\frac{3\pi a^2}{2}$ (b) $\frac{3\pi a}{4}$ (c) $\frac{3\pi a^2}{4}$ (d) $\frac{3\pi^2 a^2}{4}$

(7) If $f = y^3$, $g = x^3 + 3y^2x$ then $\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}$ = ...

(a) $3y^2$ (b) $3x^2 + 3y^2$ (c) $3x$ (d) $3x^2$

(8) Parametric form of $x^2 + y^2 = z^2$ is $\bar{r}$ =................

(a) $u\cos v\bar{i} + u\sin v\bar{j} + u\bar{k}(b)u\cos u\bar{i} + v\sin v\bar{j} + u\bar{k}$ (c) $u\cos v\bar{i} + v\sin u\bar{j} + v\bar{k}$

(d) $\cos v\bar{i} + \sin v\bar{j} + u\bar{k}$

(9) If surface $S : z = 3$ then unit normal vector $\bar{n}$ = ...

(a) 3 (b) $\bar{i}$ (c) $\bar{j}$ (d) $\bar{k}$

(10) If $\bar{n} = \bar{k}$ then dA = ...

(a) 0 (b) $dxdz$ (c) $dxdy$ (d) $dydz$

(P.T.O)

Que.2 Answer the following (Any Ten) 20

(1) Evaluate $\int_0^1 \int_x^{2x} (1 + x^2 + y^2)\, dydx$.

(2) Evaluate $\int_0^{\pi/2} \int_0^1 x^2y^2\, dydx$.

(3) Evaluate $\int_0^2 \int_0^y e^{x+y}\, dxdy$.

(4) Change the order of integration in $\int_0^c \int_0^y f(x,y)\, dxdy$.

(5) Find area of region $R : r = a(1 + \cos\theta)$.

(6) Define Line integral independent of path , Exact differential equation .

(7) Represent $\overline{r} = a\cos v \cos u\overline{i} + a\cos v \sin u\overline{j} + a\sin v\overline{k}$ in cartesian form .

(8) Represent $\overline{r} = au\cos v\overline{i} + bu\sin v\overline{j} + u^2\overline{k}$ in cartesian form .

(9) Represent $\frac{x^2}{9} - \frac{y^2}{4} = z$ in parametric form .

(10) In usual notation prove that $\iiint_R \nabla^2 f\, dv = \iint_S \frac{\partial f}{\partial n}\, dA$.

(11) Prove first and second form of Green 's theorem .

(12) Discuss Physical and Geometrical applications of triple integral.

Que.3 (a) Transform $\iint_R (x^2 + y^2)\, dxdy$ in uv-plane by taking $x + y = u$, $x - y = v$ .Then evaluate it, where R : Parallelogram with vertices $(0,0), (1,1), (2,0), (1,-1)$. 5

(b) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2\, dxdy$. 5

OR

Que.3 (c) Evaluate $\iint_R e^{-x^2-y^2}\, dxdy$ where $R : x^2 + y^2 \leq 1$. 5

(d) Let $f(x,y) = 1$ be the density of mass in region R : $0 \leq y \leq \sqrt{1-x^2}$; $0 \leq x \leq 1$, then find I_x, I_y . 5

Que.4 (a) State and prove Green's theorem for plane . 5

(b) By using Green's theorem , evaluate $\int_C [x^2y dy - xy^2 dx]$,where C : the boundary of region in the first quadrant bounded by $y = 1 - x^2$.Also check the result by direct calculation . 5

OR

Que.4 (c) In usual notation prove that $\iint_R \nabla^2 w\, dxdy = \int_C \frac{\partial w}{\partial n}\, ds$. 5

(d) Evaluate $\int_C \frac{\partial w}{\partial n}\, ds$ for $w = 2x^2 + y^2$ and C : the boundary of the region bounded by $y = x^2$ and $y = x + 2$. 5

Que.5 (a) Find area of the surface $\bar{r} = (a + b\cos v)(\cos u\bar{i} + \sin u\bar{j}) + b\sin v\bar{k}$; where $a > b > 0$, $0 \leq u, v \leq 2\pi$. 5

(b) Evaluate $\iint\limits_S f(x,y,z)\, dA$ for $f(x,y,z) = tan^{-1}(y/x)$, S : $z = x^2 + y^2$, $1 \leq z \leq 4$, $x \geq 0$, $y \geq 0$. 5

OR

Que.5 (c) State and prove divergence theorem of Gauss . 5

(d) Find area of the surface $x^2 + y^2 = a^2$, where $0 \leq z \leq b$. 5

Que.6 (a) Verify Stoke's theorem for $\bar{V} = y^3\bar{i} - x^3\bar{j}$, S : the circular disk $x^2 + y^2 \leq 1$, $z = 0$. 5

(b) State and prove Stoke's theorem . 5

OR

Que.6 (c) By using triple integral ,find moment of inertia of a mass distribution of density 1 in a region R about x-axis , where R : the cylinder $y^2 + z^2 \leq a^2$, $0 \leq x \leq h$. 5

(d) By using triple integral , find volume of the region in the first octant bounded by $x^2 + z^2 = 1$ and by the plane $y = 0, z = 0, x = y$. 5

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(3)