[28] SEAT No.________ No. of Printed Pages: 2

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SARDAR PATEL UNIVERSITY

B.Sc. SEM-III EXAMINATION (NC)

01^{st} January, 2021, Friday

10.00 a.m. to 12.00 noon

US03EMTH05

(Calculus and Algebra-1)

Maximum Marks: 70

Q.1 Choose the correct option in the following questions, mention the correct option in the answerbook. [10]

(1) Which of the following is not an indeterminate form?
(a) ∞^0 (b) $0 \times \infty$ (c) 1^∞ (d) $\frac{1}{\infty}$

(2) If $f = \sin x$ then $f_{xx} = ...$
(a) $\sin x$ (b) $\cos x$ (c) $-\sin x$ (d) $-\cos x$

(3) $z = \dfrac{x^2 y}{x^3 + xy^2}$ is a homogeneous function of degree....
(a) 0 (b) 1 (c) 2 (d) z is not homogeneous

(4) A matrix A is said to be non singular if.......
(a) $A \neq 0$ (b) $|A| = 0$ (c) $|A| \neq 0$ (d) none of these

(5) If $u = x^2 - y^2$, then $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ is
(a) 3 (b) 2 (c) 1 (d) 0

(6) $\lim\limits_{x\to 0} \left(\dfrac{\tan x}{x}\right)^{\frac{1}{x}}$ is the form of ......
(a) 0^∞ (b) 0^∞ (c) 1^∞ (d) $\frac{0}{0}$

(7) Principle diagonal entries of Skew-symmetric matrix are all....
(a) Zero (b) Zero or Pure imaginary (c) Pure imaginary (d) 1

(8) If $A = \begin{pmatrix} 4i & i-1 \\ 8i+5 & 7 \end{pmatrix}$, then $\overline{A} = ...$
(a) $\begin{pmatrix} 4i & i-1 \\ 8i+5 & 7 \end{pmatrix}$ (b) $\begin{pmatrix} -4i & -i-1 \\ -8i+5 & 7 \end{pmatrix}$
(c) $\begin{pmatrix} -4i & -i+1 \\ -8i-5 & -7 \end{pmatrix}$ (d) $\begin{pmatrix} 4i & 8i+5 \\ i-1 & 7 \end{pmatrix}$

(9) Characteristic equation of the identity matrix I of order 2 is ......
(a) $x^2 - 1 = 0$ (b) $x^2 + 1 = 0$ (c) $(x+1)^2 = 0$ (d) $(x-1)^2 = 0$

(10) A square matrix A is said to be Skew Hermitian if....
(a) $A' = A$ (b) $A' = -A$ (c) $A^\theta = A$ (d) $A^\theta = -A$

Q.2 Do as directed . [08]

(1) True or False: If A and B are any two matrices then $AB = BA$.

(2) True or False: If A and B are any two matrices then $A + B$ is always defined.

(3) True or False: If A and B are any two matrices then always $A - B = B - A$.

(4) True or False: For any matrix A, we have $(A^T)^T = A$.

(5) True or False: If A and B are any two matrices with same order then always $A = B$.

[P.T.O.]

(6) The function $f(x,y) = 3x^{18} + 11x^9y^9 + 125y^{18}$ is a homogenous function of ... degree

(7) If $u = x^2 + y^2$, then $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ is ...

(8) Is the function $f(x,y) = x^n log\left(\frac{y}{x}\right)$ is homogenous? : Yes or No. If it is yes then the degree of it is ....

Q.3 Attempt any **Ten**. [20]

(1) Evaluate: $\lim_{x\to 0} \frac{\log(\sin x)}{\cot x}$.

(2) If $A = \begin{pmatrix} 1 & -2 \\ 1 & 3 \end{pmatrix}$ then find characteristic equation of A.

(3) Evaluate: $\lim_{x\to a} (a-x)\tan\left(\frac{5\pi x}{2a}\right)$.

(4) For $u = x^3 - 3xy^2$ prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

(5) If $A = \begin{pmatrix} 3 & -1 & 2 \\ 2 & 0 & 1 \\ 1 & -1 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 & 3 \\ 6 & 2 & 1 \\ -2 & 3 & 2 \end{pmatrix}$, then obtain $A + B$.

(6) Define Hermitian matrix with an example.

(7) For $u = x\sin y + 3xy$, find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$.

(8) If U and V are two symmetric matrices, then show that UVU is also symmetric.

(9) State Reversal, Associative and Distributive law for product of matrices.

(10) Find degree of the homogeneous function $z = \frac{x^2+y^2}{\sqrt{xy}}$.

(11) If $A = \begin{pmatrix} 1 & -2 & 5 \\ 0 & 4 & 11 \\ 3 & 1 & 8 \end{pmatrix}$ then find $|A'|$.

(12) Evaluate: $\lim_{x\to 2} \frac{\sin x^2 - 4}{x-2}$.

Q.4 Attempt any **Four**. [32]

(a) Evaluate: $\lim_{x\to\infty} \left[\frac{1^{\frac{1}{x}} + 2^{\frac{1}{x}} + 3^{\frac{1}{x}}}{3}\right]^{3x}$

(b) Evaluate: $\lim_{x\to 0} \frac{e^x + \log(1-x) - 1}{\tan x - x}$.

(c) If $H = f(2x-3y, 3y-4z, 4z-2x)$ then prove that $\frac{1}{2}\frac{\partial H}{\partial x} + \frac{1}{3}\frac{\partial H}{\partial y} + \frac{1}{4}\frac{\partial H}{\partial z} = 0$.

(d) State and prove Euler's theorem for functions of two variables.

(e) Prove that Every square matrix can be expressed in one and only one way as the sum of a symmetric and skew symmetric matrix.

(f) If $A = \begin{pmatrix} -2 & -1 \\ 1 & 0 \\ 3 & -4 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 3 \\ 2 & 0 \\ -4 & -1 \end{pmatrix}$ and $2X + 3A = B$ then find X.

(g) State and prove Cayley-Hamilton theorem.

(h) If $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ then find out the values of α, β such that $(\alpha I + \beta A)^2 = A$.

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