SEAT No.________ No. of Printed Pages : 3

[73]

SARDAR PATEL UNIVERSITY

B.Sc. Sem- 3 (NC) EXAMINATION

Thursday, 07^{th} January, 2021

02.00 p.m. to 04.00 p.m.

US03EMTH01(CALCULUS)

Maximum Marks: 70

Q.1 Choose the correct option in the following questions, mention the correct option in the answerbook. [10]

(1) The integral $\int_a^\infty \frac{dx}{x^\mu}$ $(a > 0)$ is convergent if and only if....

(a) $\mu = 1$ (b) $\mu > 1$ (c) $\mu < 1$ (d) $\mu \neq 1$

(2) The integral $\int_1^2 x dx$ is....

(a) Improper integral of first kind (b) Improper integral of second kind (c) Proper integral (d) None of these

(3) The relation between β and Γ function is

(a) $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ (b) $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m-n)}$

(c) $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(mn)}$ (d) $\beta(m,n) = \frac{\Gamma(m)+\Gamma(n)}{\Gamma(m+n)}$

(4) $\Gamma(1/2) =$

(a) $1/2$ (b) π (c) $\sqrt{\pi}$ (d) 0

(5) $\Gamma(10) =$

(a) 10 (b) 10! (c) 9 (d) 9!

(6) If $f(x,y) = x^3y^2$, then $\frac{\partial^2 f}{\partial x^2}$

(a) $6x^2y$ (b) $6xy^2$ (c) $6x^2y^2$ (d) $6xy$

(7) $\nabla \cdot (\nabla \times \bar{v}) = ...$

(a) 0 (b) $\bar{0}$ (c) $\bar{v}$ (d) 1

(8) Period of $\cos x$ is....

(a) π (b) $\pi/2$ (c) 2π (d) 3π

(9) A function $f : \mathbb{R} \to \mathbb{R}$ is said to be an odd function if....

(a) $f(x) = 1$ (b) $f(-x) = f(x)$ (c) $f(-x) = -f(x)$ (d) $f(x) = 0$

(10) The function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2 + x$ is

(a) Even (b) Neither even nor odd

(c) Odd (d) Both even and odd

Q.2 Do as directed: [08]

(1) The value of $\int_1^2 x^2\, dx = \ldots.$

(2) The value of $\Gamma(5) = \ldots.$

[P.T.O.]

(3) $\int\limits_0^\infty e^{-x^2}dx. = \ldots\ldots$

(4) The smallest period (primitive period) of the function $\sin x$ is ....

(5) True or False: The integral $\int\limits_0^\infty \frac{dx}{x^\mu}$ $(a > 0)$ is convergent if and only if $\mu > 1$.

(6) True or False: $\beta(4,7) = \beta(7,2)$.

(7) True or False: $\nabla(fg) = f\nabla g + g\nabla f$.

(8) True or False: $\nabla \times (\nabla f) = \bar{0}$.

Q.3 Attempt any Ten: [20]

(1) Define : Proper and Improper integrals.

(2) Evaluate: $\int\limits_{-\infty}^{\infty} \frac{1}{x^2}\, dx$.

(3) Evaluate: $\int\limits_{1}^{\infty} \frac{1}{\sqrt{x}}\, dx$.

(4) Prove that $\beta(m,n) = \beta(n,m)$.

(5) Prove that $\frac{\beta(p+1,q)}{p} = \frac{\beta(p,q)}{p+q}$.

(6) Prove that $\int\limits_0^\infty \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}}\, dx = 2\beta(m,n)$.

(7) Define Divergence and Curl of a vector field.

(8) Prove that $\nabla(fg) = f\nabla g + g\nabla f$.

(9) Prove that $\nabla(f \pm g) = \nabla f \pm \nabla g$.

(10) Define Periodic function and Period of function.

(11) Find the primitive period of the function $f(x) = \sin mx$.

(12) Write the fourier series and fourier coefficients for an odd function.

Q.4 Attempt any Four: [32]

(a) Prove that the integral $\int_a^b \frac{dx}{(x-a)^{\mu}}$ is convergent if and only if $\mu < 1$.

(b) Examine the convergence of $\int_0^1 \frac{x^{(a-1)}dx}{(1+x)}$.

(c) Prove that $\frac{\beta(p,q+1)}{q} = \frac{\beta(p+1,q)}{p} = \frac{\beta(p,q)}{p+q}$.

(d) Prove that $\int_0^{\infty} \frac{x^{m-1}dx}{(a+bx)^{m+n}} = \frac{1}{a^n b^m}\beta(m,n)$.

(e) Prove that (i) $\nabla.(f\bar{v}) = f(\nabla.\bar{v}) + \bar{v}.\nabla f$ (ii) $\nabla \times (\nabla f) = \bar{0}$.

(f) Find direction derivative of $f(x,y,z) = 2x^2 + 3y^2 + z^2$ at point (2,1,3) in the direction of $\bar{a} = \bar{i} - 2\bar{k}$.

(g) Find the Fourier coefficient of the following periodic function $f(x) = \begin{cases} 1 & \text{if } -\pi/2 < x \leq \pi/2 \\ -1 & \text{if } \pi/2 < x < 3\pi/2. \end{cases}$

(h) Represent the function $f(t) = 1$, $0 < t < l$ by Fourier cosine series.