

SEAT No. _____

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SARDAR PATEL UNIVERSITY
B.Sc Semester – III (CBCS) Examination

Monday, 4th January 2021

Time : 2 to 4 pm

Statistics

M.Marks : 70

US03CSTA22 (Elements of Probability Theory)

- Q.1 Multiple Choice Questions (10 × 1)
- 1 Given $P(A \cap B) = 3/5$ then $P(A^c \cup B^c) =$ _____
(a) 2/5 (b) 3/5 (c) 1/5 (d) 1
- 2 $P(X < 7) =$ _____ if the pdf of X is
$$f(x) = \begin{cases} \frac{x}{25}, & 0 < x < 5 \\ \frac{10-x}{25}, & 5 < x < 10 \\ 0, & \text{otherwise} \end{cases}$$

(a) 31/50 (b) 41/50 (c) 91/50 (d) none of these
- 3 If $M_X(t) = (1 - 3t)^{-1}$ then $V(1 + 4X) =$ _____
(a) 13 (b) 144 (c) 12 (d) 81
- 4 If the two r.v's X and Y are independent
(a) The joint distribution equal to product of their marginal distributions
(b) The conditional distribution of X given Y equal to marginal distribution of X
(c) Their covariance is zero
(d) All of the above are correct
- 5 Two balls are drawn at random with replacement from a box containing 10 black and 6 red balls. The probability that first ball is black and second is red _____
(a) 64/15 (b) 1/8 (c) 25/64 (d) 15/64
- 6 Two dice are rolled. What is the probability of getting two numbers whose product is prime number?
(a) 1/6 (b) 1/12 (c) 3/8 (d) 5/16
- 7 If $f(x, y) = (x^2 + y^2)/112, x, y = 0, 1, 2, 3$, is the joint probability distribution of X and Y then $P(X = 2) =$ _____
(a) 14/112 (b) 18/112 (c) 30/112 (d) None of these
- 8 Let X has a p.d.f. $f(x) = 2(1-x), 0 < x < 1$ and zero otherwise. What is the median?
(a) $1/\sqrt{2}$ (b) $1 + (1/\sqrt{2})$ (c) $1 - (1/\sqrt{2})$ (d) $\sqrt{2} - 1$
- 9 Given that $f(x) = k(1+x)^2, x = 0, 1, 2, 3$ and zero otherwise. What will be the value of k so that given $f(x)$ being p.m.f?
(a) 2/30 (b) 1/30 (c) 3/30 (d) 4/30
- 10 Let X be a r.v. with probability distribution
 $f(x) = 2/3^x, x = 1, 2, \dots$ and zero otherwise then probability that X is odd
(a) 1/4 (b) 1/2 (c) 3/4 (d) 1/8

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[P.T.O.]

- (4 × 1)
- Q.2 Fill in the blanks**
- 1 Let X and Y be two random variables with joint pdf
 $f(x, y) = c(2x + y), 0 < x < 1, 0 < y < 2$ and zero otherwise, then $c =$ _____
 - 2 If X and Y are two independent r.v.'s with $V(X) = 5$ and $V(Y) = 10$ then
 $V(2X + Y) =$ _____
 - 3 Let a r.v. X denote the waiting time of a passenger at a railway station then X is a
 _____ type of r.v.
 - 4 If A and B are two mutually exclusive and exhaustive events with $P(A) = 2P(B)$ then
 $P(A) =$ _____
True - False (4 × 1)
 - 5 The probability of occurrence of at least one of two events is the sum of the probability of each of the two events.
 - 6 If X and Y are two independent r.v.'s then they are uncorrelated.
 - 7 The m.g.f of sum of two random variables is the product of their m.g.f's.
 - 8 If A and B are two events then $P(A \cap B^c) = P(B) - P(A \cap B)$
- (10 × 2)
- Q.3 Short Type Questions (Attempt Any Ten)**
- 1 Define moment generating function. State its properties.
 - 2 Examine whether the following is a p.m.f. or not?
 $P(x) = 1/(2x), x = 1, 2, \dots$ and zero otherwise
 - 3 Examine whether $f(x, y) = 6x^2y, 0 < (x, y) < 1$ and zero otherwise, is joint pdf of X and Y or not.
 - 4 An urn contains 7 white and 3 red balls. Two balls are drawn at random, without replacement from this urn, find the probability distribution of X where $X =$ no. of red balls drawn.
 - 5 State and prove additive law of probability for two events.
 - 6 If X and Y are two independent r.v.'s then show that $V(X + Y) = V(X - Y)$
 - 7 The joint pdf of two r.v.'s X and Y is
 $f(x, y) = c(x^2 + y^2), 0 < (x, y) < 1$ and zero otherwise. Determine the constant c .
 - 8 If $f(x) = 3x^2, 0 < x < 1$ and zero otherwise, is the pdf of X . Find Inter Quartile Range.
 - 9 Prove that the complementary events of two independent events are also independent.
 - 10 If $f(x) = 1/4, -2 < x < 2$ and zero otherwise, is the pdf of X . Show that all the odd order moments are zero.
 - 11 If $V(X + Y) = 3, V(X - Y) = 1, E(X) = 1, E(Y) = 2$ then find $E(XY)$.
 - 12 If X and Y have the joint pdf $f(x, y) = \frac{3}{4} + xy, 0 < (x, y) < 1$ and zero otherwise
 Find the conditional distribution of Y given $X = x$.
 (4 × 8)
- Q.4 Long Answer Questions (Attempt Any Four)**
- (a) A continuous random variable X in the range $(-3, 3)$ is given by the pdf

$$f(x) = \begin{cases} \frac{1}{16}(3+x)^2, & -3 \leq x < -1 \\ \frac{1}{16}(6-2x^2), & -1 \leq x \leq 1 \\ \frac{1}{16}(3-x)^2, & 1 \leq x \leq 3 \end{cases}$$

- (i) Verify that the area under the curve is unity (ii) Find the distribution function of X
 (iii) $E(2X)$ (iv) $P(1/2 < X < 2)$.
- (b) (i) The coefficient of correlation between X and Y is $1/3$, $V(X) = K$, $V(Y) = 4K$ and $V(Z) = 114$, Where $Z = 3X - 4Y$. Find the constant K .
 (ii) Let X and Y have the joint prob. mass function
 $f(x, y) = k(x + y)$, $x = 1, 2, 3$; $y = 1, 2$ and zero otherwise
 (a) Determine the value of k (b) the conditional distribution of Y given X (c) $P(X + Y > 3)$
- (c) (i) Let A and B be two events such that $P(A) = P(B) = 1/2$ and $P(A^c \cap B^c) = 1/3$ then find probability of exactly one of two event occurs.
 (ii) If $P(A) = 0.25$ and $P(B) = 0.8$ then show that $0.05 \leq P(A \cap B) \leq 0.25$

(d) Following are the compositions of two basket flowers:

Basket - I	Basket - II
Pink - 5	Pink - 4
White - 3	White - 5
Yellow - 2	Yellow - 4

One basket is chosen at random and two flowers drawn from it they happen to be white and yellow. What is the probability that they come from basket - I, II?

(e) (i) Let X be a r.v. with the following probability distribution:

x	0	1	2	3	4	5	6
$P(x)$	$1/20$	P_1	$1/5$	P_2	P_3	$1/10$	$1/10$

If $E(X) = 3.1$, $E(X^2) = 12.1$. find P_1 , P_2 and P_3 .

(ii) There are 5 cards numbered 1 to 5, one number on one card. Two cards are drawn at random without replacement. Let X denotes the sum of the numbers on two cards drawn. Find the mean and variance of X .

(f) The probability distribution of a r.v. X is given below:

x	1	2	3	4	5	6	7	8
$P(x)$	k	k	$3k$	$2k$	$k^2 + k$	$2k^2$	$4k^2 + k$	$3k^2$

Find (i) k (ii) the c.d.f. of X (iii) the minimum value of k so that $P(X \leq k) = 1/2$

(iv) $P(2 \leq X < 7)$ (v) $P(X > 3)$.

(g) The joint p.d.f. of X and Y be

$$f(x, y) = k(x^2 + y^2), 0 < x < 1, 0 < y < 1 \text{ and zero otherwise,}$$

(i) Find the value of k (ii) the correlation coefficient between X and Y .

(h) (i) Two independent r.v's are uncorrelated but converse is not always true. Justify your answer by giving counter example.

(ii) The moment generating function of X is $M_X(t) = (4 - 3e^t)^{-1}$, find the mean and variance of X .

————— X —————

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