SEAT No._____ No. of Printed Pages : 3

[32]

Sardar Patel University
S.Y.B.Sc.(Semester-III) Examination
Subject:-Elements of Probability theory
Paper Code:-US03CSTA02

Date:-04/01/2021,Monday Time:-10:00 a.m. to 12:00 p.m. Marks:-70

Q.1. Multiple Choice Questions: - [10]

1 For the events A and B if $A \subset B$ then the relation between P(A) and P(B) is _____.
(a) $P(A) \leq P(B)$ (a) $P(A) \geq P(B)$ (c) $P(A) \neq P(B)$ (d) none.

2 If two fair dice are tossed then the probability that sum is 7 = ____.
(a) 8 /36 (b) 7 /36 (c) 6 /36 (d) 5 /36

3 If $F(x) = 0, x < 0$
$= x^2, 0 \leq x < 1$
$= 1, x \geq 1$, is the c.d.f. of the random variable X then f(x) =_____.
(a) 2x (b) $3x^2$ (c) $x^3/3$ (d) none.

4 The out- come of a random experiment of tossing of three fair coins is a random variable of _____ type.
(a) continuous (b) Discrete (c) both (a) & (b) (d) none.

5 If f(x) is the p.d.f of the continuous random variable X and if $P(X \leq M) = P(X \geq M)$ then M = _____.
(a) mean (b) median (c) mode (d) none.

6 For any type of random variable X, V(X) = ____.
(a) $E(X - A)^2$ (b) $E(X - E(X))^2$ (c) $E(X + E(X))^2$ (d) none.

7 V(X) = 16 then V(2X - 8) =_____.
(a) 32 (b) 64 (c) 72 (d) 24

8 If X and Y are two independent random variables with E(X) = 9 and E(X.Y) = 36 th E(Y) = _____.
(a) 6 (b) 4 (c) 2 (d) none.

9 If for two random variables X and Y, E(X) = 5 , E(Y) = 4, V(X) = 25 , V(Y) = 16 and E(X.Y) = 5then r _____.
(a) 0.75 (b) - 0.75 (c) -1 (d) none.

10 If the conditional p.d.f. of X given Y = y is equal to the marginal p.d.f. of X then the two random variables X and Y are _____ variables.
(a) continuous (b) Dependent (c) Independent (d) none.

Q.2. Fill in the blanks with appropriate answer :- [08]

1 The interval (Q_2, Q_3) includes _____ % of points.

2 Four coins are tossed, the number of sample points in a sample space is_____.

3 If F(x) is the c.d.f. of a continuous random variable X then the p.d.f. f(x) = ___

4 If $f(x) = 2x, 0 < x < 1$
$= 0$, otherwise, is the p.d.f. of the random variable X then $P(X \leq 1)$ = ___.

State whether following are true or false.

5 Cov(X,Y) = 0 => X and Y are dependent.

6 If V(X)= 1, then V(2X -3) is 1.

7 If the conditional p.d.f. of X given Y = y is equal to the marginal p.d.f. of X then the two random variables X and Y are independent variables.

8 If f(x,y) is the joint p.d.f. of two independent random variables X and Y, then f(x,y) = f(x)+f(y) .

Q.3. Short Questions: - (Attempt any Ten) [20]

1 Write an appropriate sample space for tossing of two fair dice. Find the probability for the following events. (i) the sum is greater than 10.
(ii) the first die shows an even number. (iii) the sum is either five or seven.

2 Prove in usual notations for any event $A \subset S$ that P (A') = 1 - P (A).

3 Prove in usual notations for three events A, B and C of sample space S, that $P(A \cup B / C) = P(A / C) + P(B / C) - P(A \cap B / C)$.

4 Consider the experiment of tossing of three fair coins. Let variable X denote the number heads. Find the pmf and cdf.

5 Define the probability density .

6 If $f(x) = 2x, 0 < x < 1$
$= 0$, otherwise, is the p.d.f. of the random variable X then find c.d.f. of X and hence find $P(0.2 \leq X \leq 0.8)$.

7 Define Mathematical Expectation, Moment generating function

8 Let X be a r.v.with the following prob. distribution:

x	-3	6	9
p(x)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Find E(x) and $E(x^2)$.

9 Two unbiased dice are thrown. Find the expected value of sum of the numbers on upper face of dice.

10 Define the joint probability distribution, marginal probability distribution of x and y.

11 If $f(x,y) = \frac{x+y}{21}$, x=1,2,3; y = 1,2
$= 0$, elsewhere.
Find (i)$P(X \leq 1)$(ii) $P(X \leq 2, y \leq 1)$

12 Is $f(x,y) = \frac{2x+3y}{120}$, x=1,2,3; y = 1,2
$= 0$, elsewhere is the joint pmf of X and Y?

Q.4. Long Questions: - (Attempt any four) [32]

1 Prove in usual notation that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Hence (i)prove that $P(A \cap B) \geq P(A)+P(B) -1$.(ii) obtain the result for three events A,B and C.

2 (a)A Die is thrown. Find the prob. of getting
(i)An even number (ii) a prime number (iii) a number greater than or equal to 3 (iv) a number more than 6 (v) a number less than equal to 6 (vi) a number less than 5 (vii) number more than 1 (viii) number less than 1.

3 If $f(x) = kx, 0 < x < 2$
$= k(5-x), 2<x< 5$
$= 0$, otherwise is the pdf of x then find (i) k (ii) $P(1 <x \leq 2)$
(iii) $P (1\leq x \leq 3)$ (iv)$P(x\geq 4)$. (v) $P(x <1)$ (vi) $P(x <4)$ (vii) $P(2 <x<4)$
(viii) $P(3<x<5)$

4 A discrete r.v. has the pmf

x	1	2	3	4	5	6	7
p(x)	k	2k	2k	3k	$2k^2 + k$	$3k^2$	$5k^2$

Determine k (ii) $P (X \leq 4)$ (iii) $P (2< X < 7)$
(iv) Find the min. value of a such that $P (X\leq a) < 0.5$.
(iv) $P(3< X < 7/ X> 4)$. (v) $P(2<X<4/ X>3)$ (vi) $P(X >6)$ (vii) $P(1<X<5)$ (viii) $P(2<X<4)$.

5 If $Mx(t) = e^{25t(1+t)}$ is the mgf of a r.v X then find first four moment of X and hence find β_1 and β_2.

6 If $Mx(t) = (\frac{2}{3} + \frac{1}{3} e^t)^9$ is the mgf of a discrete rv X then find E(X), V(X) E(2X+3) V(2X+3). Also obtain β_1 and β_2.

7 For the bi variate prob. distribution of X and Y find
(i)$P (X\leq 1, Y =2)$ (ii) $P (X\leq 1)$ (iii) $P (Y\leq 3)$ (iv) $P (X < 3, Y \leq 4)$ (v) the marginal distribution of X and Y (vi) conditional distribution of X given Y =1
(vii) conditional distribution of Y given X =1.

X\ Y	1	2	3	4	5	6

0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{2}{64}$	$\frac{2}{64}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$

8 If f(x,y) =k(3x+2y), x=1,2,3,4; y = 1,2,3
= 0, elsewhere is the joint pmf of X and Y. Find (i)k (ii)the marginal distribution of X and Y. (iii) P(X< 1,Y<2).

———— X ————