Seat No. : ________

SARDAR PATEL UNIVERSITY

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B.Sc. (III-Semester) EXAMINATION 2021

Friday, 1st January, 2021

02:00pm- 04:00pm

US03CMTH 22-Mathematics

MULTIVARIATE CALCULUS

Total Marks: 70

Note: Figures to the right indicates full marks of question.

Q: 1 Answer the following by selecting the correct answer from the given options: **[10]**

1. If $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$ is a constant vector and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ then $div(\vec{a} \times \vec{r}) =$ -----------

   a. 0 b. -1 c. 1 d. a

2. The greatest value of the directional derivative of $f = e^{x+y+z}$ at point $(1,0,-1)$ is -------

   a. 1 b. 3 c. 2 d. 4

3. The value of $\int_0^\infty x^2e^{-x}dx$ is ----------

   a. 0 b. 2 c. -2 d. ∞

4. $\int_0^3 \int_0^y dxdy =$ --------------

   a. 3/2 b. $-9/2$ c. 9/2 d. 3

5. If we change Cartesian variable (x, y) to polar variable (r, θ) in double integration then dx dy = ----

   a. $drd\theta$ b. $rdrd\theta$ c. $|J|\, drd\theta$ d. $r^2drd\theta$

6. In double integration total mass M of density 1 over region $0 \le x \le 2; 0 \le y \le 1$ is ------

   a. 1 b. 0 c. 4 d. 2

7. The surface $\bar{r} = acosvcosu\vec{i} + acosvsinu\vec{j} + asinv\vec{k}$ represents --------

   a. sphere b. elliptic paraboloid c. circle d. hyperbolic paraboloid

8. If $yzdx + xzdy + xydz = ----$

   a. $d\left(\frac{x^2+y^2+z^2}{2}\right)$ b. $d(xyz)$ c. $d(x^2 + y^2 + z^2)$ d. $d\left(\frac{xyz}{2}\right)$

9. $\int_0^1 \int_0^1 \int_0^1 xyzdxdydz =$ --------

   a. 8 b. 1/8 c. 1/2 d. 1/4

10. A function f(x, y, z) is said to be harmonic if $\nabla^2 f = ---$

    a. 1 b. 2 c. -1 d. 0

Q: 2 Do as Directed: **[08]**

1. True or False: The vector $3x^2\bar{i} - 4y\bar{j} + z\bar{k}$ is irrotational.
2. True or False: If $\emptyset = xyz$ then value of $|grad\emptyset|$ at the point (1,2,-1) is 3.
3. True or False: Work done by force $\bar{P}$ over the curve C is given by $W = \int_C \bar{P}d\bar{r}$
4. True or False: The moment of inertia about origin defined as $I_0 = I_x - I_y$
5. $\int_0^a \int_0^{\sqrt{a^2-x^2}} dxdy = -------$
6. The area of plane region in polar form is given by A=-----------
7. In Stock's theorem $\int\int_S (\bar{\nabla} \times \bar{V}).\bar{n}dA = -------$
8. The first form of Green's theorem is $\int\int_R \int[f\bar{\nabla}^2 g + \bar{\nabla}f.\bar{\nabla}g]dV = -----$

Q: 3 Answer in brief of the following questions. ***(Any Ten)*** **[20]**

1. In usual notation prove that $\beta(m,n) = \beta(n,m)$
2. For which value of a the surface $ax^2 - yz = (a+2)x$ and $4x^2y + z^3 - 4 = 0$ are perpendicular to each other at the point $(1,-1,2)$?
3. Find the equation of tangent plane and normal line to the surface $x^3y^2 - 3x^2z^3 = -zy + 2$ at the point (0,2,1).
4. Evaluate $\int_C (y^2dx - x^2dy)$ where C: along the straight line from (0,1) to (1,0).
5. Evaluate: $\int_0^2 \int_0^y e^{x+y}dxdy$
6. Change the order of integration $\int_0^c \int_0^y f(x,y)dxdy$
7. Verify whether the line integral $\int_{(1,1,2)}^{(3,-2,-1)}(yzdx + xzdy + xydz)$ is independent of path or not. Also calculate.
8. By using line integral find the area of region $r = a(1+cos\theta)$.
9. For which value of u $\bar{r}_u \times \bar{r}_v = 0$? where $\bar{r} = ucosv\bar{i} + usinv\bar{j} + u\bar{k}$.
10. Evaluate $\int\int_S (sinxdydz + y(2-cosx)dzdx)$ where $S: 0 \le x \le 3, 0 \le y \le 2, 0 \le z \le 1$ by applying divergence theorem.
11. Let R be a closed region in space and S be its boundary. Let g be harmonic function in R, then prove that $\int\int_S \frac{\partial g}{\partial n}dA = 0$
12. If $\bar{V} = \bar{\nabla}f$ then prove that $\int_C \bar{V}_t ds = 0$.

Q: 4 Attempt any Four of the following. **[32]**

(1) State and prove Duplication formula.

(2) In usual notation prove that: (i) $curl(r^n\bar{r}) = \bar{0}$

(ii) $\nabla^2(r^n\bar{r}) = n(n+3)r^{n-2}\bar{r}$ Where $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ and $r = |\bar{r}|$

(3) Let $f(x,y) = 1$ be the density of mass M in region $R: 0 \le y \le \sqrt{1-x^2},\ 0 \le x \le 1$ then find centre of gravity and moment of inertia I_x, I_y & I_o.

(4) Transform $\int\int_R (x-y)^2 sin^2(x+y)dxdy$ in uv-plane by taking $u = x - y, v = x + y$ where R is the parallelogram with vertices $(\pi, 0), (2\pi, \pi), (\pi, 2\pi)$ &$(0, \pi)$. Hence evaluate it.

(5) State and prove Green's theorem for plane.

(6) Verify both vector form (divergence and curl form) of Green's theorem for the given $\bar{V}$ and C, $\bar{V} = y\bar{i} + 4x\bar{j}$, C: the boundary of triangle with vertices (0,0), (2,0) & (2,1).

(7) State and prove Divergence theorem of Gauss.

(8) Verify the Stock's theorem for $\bar{V} = y^3\bar{i} - x^3\bar{j}$; and surface S: the circular disk $x^2 + y^2 \le 1, z = 0$

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