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[103]

SARDAR PATEL UNIVERSITY

B.Sc. SEM- III EXAMINATION

31st December 2020 , Thursday

02:00 p.m. to 04:00 p.m.

Sub.: Mathematics (US03CMTH21)

(Numerical Methods)

Maximum Marks: 70

Q.1 Choose the correct option in the following questions, mention the correct option in the answerbook. [10]

(1) In iteration method the sequence of approximation converges to the root provided
(a) $|\phi'(x)| > 1$ (b) $|\phi'(x)| < 1$ (c) $|\phi(x)| \neq 1$ (d) $|\phi(x)| < 1$

(2) The root of the equation $f(x) = x^3 - 2x - 5 = 0$ lies between
(a) 0 and 1 (b) -2 and -1 (c) 2 and 3 (d) 1 and 2

(3) Central difference operator $\delta_{y1/2}$=
(a) $y_1 - y_0$ (b) $y_{3/2} - y_{1/2}$ (c) $y_{5/2} - y_{3/2}$ (d) none

(4) If $y_5 = 4$ and $y_{15} = 10$ then $E^5 y_{10} =$
(a) 10 (b) 5 (c) 15 (d) 20

(5) In forward differences $\Delta^2 y_0 =$ ....
(a)$\Delta y_0 - \Delta y_1$ (b)$y_2 - 2y_1 + y_0$ (c)$y_2 + 2y_1 + y_0$ (d)$\Delta y_2 - \Delta y_0$

(6) Name of the formula $y_p = y_0 + p\Delta y_{-1} + \frac{p(p+1)}{2!}\Delta^2 y_{-1} + \frac{p(p+1)(p-1)}{3!}\Delta^3 y_{-2} + \frac{p(p+1)(p-1)(p-2)}{4!}\Delta^4 y_{-2} + \ldots$ is
(a) Newtons Forward (b) Striling (c) Gauss Forward (d) Gauss Backward

(7) By putting $n =$ .... in the General formula for integration, we get simpson's $\frac{1}{3}$ rule.
(a) 1 (b) 2 (c) 3 (d) -1

(8) Trapezoidal rule is obtain from.....
(a) Simpson's $\frac{1}{3}$ rule (b)General formula for integration
(c) Simpson's $\frac{3}{8}$ rule (d)Gauss formula

(9) The Lagrange's interpolation formula is applicable for..........arguments.
(a) Equal spaced (b) Unequal spaced (c) positive (d) negative

(10) In general $[x_1, x_2] =$
(a) $[x_4, x_3]$ (b) $[x_3, x_3]$ (c) $[x_2, x_1]$ (d) None of these

Q.2 Do as directed. [08]

(1) True or False: The divided differences are not symmetrical in their argument.
(2) True or False: $\Delta + \nabla = \nabla\Delta$.
(3) True or False: In the Newtons backward fromula, $p = \frac{x-x_0}{h}$.
(4) True or False: The Lagranges interpolation fromula is only useful for unequal length of arguments.
(5) If $y_6 = 10$ and $E^n y_2 = 10$ then $n = \ldots$.
(6) $x_5 = x_4 - \frac{f(x_4)}{f'(x_4)}$ is approximation of .... method.
(7) The interval $[-2, 2]$ is divided into 8 equal subintervals then the length of each subinterval is ....
(8) ... is an interval containing an initial approximation for the root of an equation $x^3 - 4x - 9 = 0$.

Q.3 Attempt **any ten** in short: [20]

(1) Define averaging and shift operators.

[P.T.O.]

(2) State Simpson's $\frac{1}{3}$ rule.

(3) Prove that $\Delta - \nabla = \delta^2$.

(4) Discuss the method of successive approximations in short.

(5) If $\phi(x) = (2x+5)^{(1/3)}$ then find $|\phi'(x)|$ for x=2.5 .

(6) In usual notations show that $\Delta - \nabla = \Delta\nabla$.

(7) State Newton's forward interpolation formula for equally spaced arguments.

(8) Explain interpolation.

(9) If $y_0 = 3, y_1 = -2, y_2 = 0, y_3 = 5$ then find $\Delta^3 y_0$.

(10) Define divided differences.

(11) State Lagrange's interpolation formula for unequal intervals.

(12) Derive a formula to calculate the cube root of a number N.

Q.4 Attempt any Four: [32]

(a) Obtain the real root of the equation $x^3 + x^2 - 1 = 0$ correct upto three decimal points using bisection method.

(b) Obtain the real root of the equation $x^3 - 4x - 9 = 0$ correct upto three decimal points using false position method.

(c) Using Gauss's forward interpolation formula find $f(32)$, given that $f(25) = 0.2707, f(30) = 0.3027, f(35) = 0.3386, f(40) = 0.3794$.

(d) Derive Newton's backward interpolation formula for equally spaced values of argument.

(e) Using Lagrange's interpolation formula express the function $\dfrac{x^2+x-3}{x^3-2x^2-x-2}$ as sums of partial fraction.

(f) Obtain first and second order numerical differentiation formula for Newton's forward difference formula.

(g) Derive the general formula for numerical integration.

(h) Evaluate $\int_0^1 \frac{1}{1+x}dx$, correct to three decimal places using Trapezoidal rule and by taking $h = 0.05$.

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