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B.Sc. (III-Semester) EXAMINATION 2021(NC)

Thursday, 07thJanuary

10:00am-12:00pm

US03CMTH 02-Mathematics

NUMERICAL ANALYSIS

Total Marks: 70

Note: Figures to the right indicates full marks of question.

Q: 1 Answer the following by selecting the correct answer from the given options: **[10]**

1. In usual notation the formula $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$ is used in ---------- method.

   a. False position b. Bi-section c. Iteration d. N-R

2. For the function $f(x) = x^3 - x - 4 = 0$ the root of an equation lies between ----------

   a. $(1, 2)$ b. $(-1, 0)$ c. $(2, 3)$ d. $(0, 1)$

3. If f(a) and f(b) are opposite in signs then the graph of y=f(x) must intersect --------- in at least one point.

   a. line $y=x$ b. line $y=-x$ c. Y-axis d. x-axis

4. In Back- difference $\nabla^2 y_2 =$ ----------

   a. $\nabla y_2 - \nabla y_1$ b. $y_2 + 2y_1 + y_0$ c. $y_2 - 2y_1 + y_0$ d. $\nabla y_2 - \nabla y_0$

5. What is a linear polynomial which takes the values y(0)=1 and y(1)=0 --------?

   a. $1 - x$ b.$1 + x$ c. $2 - x$ d. $2 + x$

6. If $\nabla y_{10} = 10$ and $y_{10} = 25$ then y_9 =---------

   a. 15 b. -15 c. 5 d. -5

7. The Lagrange's interpolation formula is applicable for --------- arguments.

   a. Equi-spaced b. Un Equi-spaced c. Positive d. Negative

8. Taking mean of Gauss's Forward and Gauss's Back-ward one can obtain --------

   a. Stirling's Formula b. Bessel's Formula c. Everett's Formula d. None

9. Trapezoidal rule is obtain from --------

   a. Simpson's 1/3 rule b. Simpson's 3/8 rule c. Gauss formula d. General formula for Integration.

10. For $\int_0^6 \frac{dx}{1+x}$ in Simpson's 1/3 rule with 4 strips, then h=-------- ---------

    a. 1.5 b. 1 c. 0.5 d. 2

[P. T. O.]

Q: 2 Do as directed: **[08]**

(1) $cosx + 2 = 3x$ is know as ----------equation.

(2) For a function $f(x)$ if $f(a) < 0$ and $f(b) > 0$ then, there exist ---------- $x\epsilon(a, b)$ such that $f(x)=0$.

(3) True or False: For Forward and Backward difference $\Delta y_4 = \nabla y_5$.

(4) True or False: For the relation $y_{n+4} = E^{-2}y_{n-2}$.

(5) True or False: Divided differences are symmetric of their arguments.

(6) True or False: Newton's Forward formula for Equi-space of argument is a general case of Newton's divided difference formula.

(7) $y^{(n)} = y_0 + \int_{x_0}^{x} f(x, y^{(n-1)})\, dx$ is known as ---------- formula.

(8) By putting n=---------- in the General formula for integration, we get Simpson's 3/8 rule.

Q: 3 Answer in brief of the following questions. *(Any Ten)* **[20]**

1. If $x_1 = 1.5,\ x_2 = 1.535369,\ x_3 = 1.515710, \Delta x_1 = 0.035369, \Delta x_2 = -0.019659$, then find x_4 by using Aitkin's Δ^2 – process.
2. Find first approximation of a root of $x^3 + 8x - 7 = 0$ using Bi-section method.
3. Derive the formula to obtain $\sqrt{N}$ by using N-R method.
4. In central difference interpolation formula, when the difference table ends with odd difference then which is the suitable formula? What is a range of u?
5. Without preparing difference table find $\Delta^4 y_0$ where $y_0 = 1, y_1 = 11, y_2 = 21, y_3 = 28, y_4 = 29$
6. In usual notation prove that $\Delta - \nabla = \Delta\nabla$
7. Construct divide difference table from the given data:

x	1	3	4	6
y	-3	9	30	132

8. Using Lagrange's interpolation formula, find $f(9)$ for the data:

x	2	4	7
y	10	26	65

9. Obtain the value of first approximation in method of Successive approximation.
10. State second order Runge-Kutta formula.
11. By using Trapezodial rule, evaluate $\int_0^2 f(x)\, dx$, where $f(0) = 4, f(1) = 30, f(2) = 19$ taking n=2
12. Given that $\frac{dy}{dx} = x + y^2$, using Euler modified method, obtain $y_1^{(1)}$ taking $y_1^{(0)} = 1.1$ and h=0.1

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Q: 4 Attempt any Four of the following:

(1) Discuss the Aitken's Δ^2 – process for approximation of a real root of an equation.

(2) Find a real root of an equation $cosx = 3x - 1$ by Iteration method correct up to three decimal places.

(3) Derive Newton's backward difference interpolation formula for equally spaced values arguments.

(4) By using Gauss's backward interpolation formula find cubic polynomial f(x), given that f(1)=-1, f(2)=11, f(3)=35, f(4)=77, f(5)=143. Hence find f(0).

(5) Obtain Newton's divided difference formula for un-equi spaced values of arguments.

(6) The following table of x and y is given find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when x=1.

x	1	1.2	1.4	1.6	1.8	2	2.2
y	2.7183	3.32.01	4.0552	4.9530	6.0496	7.3891	9.0250

(7) Derive Simpson's $\frac{3}{8}$ rule for Numerical Integration.

(8) Use Picard's method to approximate y when x=0.1 given that $y(0) = 1$ and $\frac{dy}{dx} = \frac{y-x}{y+x}$ correct up to three decimal places.