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SARDAR PATEL UNIVERSITY

B.Sc.(SEMESTER - III) EXAMINATION (NC)(OLD COURSE)

Wednesday , 6^{th} Jan, 2021

MATHEMATICS : US03CMTH01

(ADVANCED CALCULUS)

Time : 2 : 00 to 4 : 00 Pm Maximum Marks : 70

Que.1 Choose the correct option to fill the following blanks . [10]

(1) For the curve $y = -x$, $\frac{ds}{dt} =$ ....
(a) 2 (b) 1 (c) $\sqrt{2}$ (d) $-\sqrt{2}$

(2) If region R is represented by $a \le x \le b$; $f_1(x) \le y \le f_2(x)$ then $\iint_R f(x,y)\,dxdy =$ ....
(a) $\int_a^b \int_{f_1(x)}^{f_2(x)} f(x,y)\,dydx$ (b) $\int_a^b \int_c^d f(x,y)\,dxdy$ (c) $\int_{f_1(x)}^{f_2(x)} \int_a^b f(x,y)\,dxdy$ (d) $\int_a^b \int_{f_1(x)}^{f_2(x)} dydx$

(3) $\int_0^1 \int_0^2 dxdy =$ ....
(a) 1 (b) 0 (c) 3 (d) 2

(4) Area of plane region in Cartesian form is given by A = ...
(a) $\frac{1}{2}\int_C [x\,dx + y\,dy]$ (b) $\frac{1}{2}\int_C [x\,dy - y\,dx]$ (c) $\frac{1}{2}\int_C [x\,dy + y\,dx]$ (d) $\frac{1}{2}\int_C [x\,dx - y\,dy]$

(5) Area of limacon $r = a(1 + \cos\theta)$ in the first and second quadrant is A = ...
(a) $\frac{3\pi a^2}{2}$ (b) $\frac{3\pi a}{4}$ (c) $\frac{3\pi a^2}{4}$ (d) $\frac{3\pi^2 a^2}{4}$

(6) If $\bar{v} = 7x\bar{i} - 3y\bar{j}$ then $\iint_R (\nabla \times \bar{v}) \cdot \bar{k}\ dxdy =$ ....
(a) 1 (b) 2 (c) -1 (d) 0

(7) If $\bar{r} = u\,\bar{i} + v\,\bar{j} + uv\,\bar{k}$ then $EG - F^2 =$ ....
(a) $1 + v^2$ (b) uv (c) $1 + v^2 + u^2$ (d) $1 + u^2$

(8) Parametric form of the plane $y = x$ is $\bar{r} =$................
(a) $u\bar{i} + v\bar{j} + u\bar{k}$ (b) $u\bar{i} + u\bar{j} + v\bar{k}$ (c) $v\bar{i} + u\bar{j} + v\bar{k}$ (d) $u\bar{i} + \bar{j} + v\bar{k}$

(9) $\int_0^1 \int_0^1 \int_0^1 x\,dxdydz =$ ....
(a) 1 (b) 0 (c) 2 (d) 1/2

(10) If $\bar{n} = \bar{k}$ then $dA =$ ....
(a) 0 (b) $dxdz$ (c) $dxdy$ (d) $dydz$

Que.2 Fill in the blanks. [8]

(1) For $x + y = u$, $x - y = v$, Jacobian J = ...

(2) In double integral , Total mass M of density 1 over region $0 \le x \le 2$; $0 \le y \le 1$ is ....

[P.T.O.]

(3) $yzdx + xzdy + xydz =$ ....

(4) If $f = y^3$, $g = x^3 + 3y^2x$ then $\frac{\partial f}{\partial y} - \frac{\partial g}{\partial x} =$ ....

(5) Parametric form of $x^2 + y^2 = z^2$ is $\bar{r} =$................

(6) If $\bar{r} = u\,\bar{i} + v\,\bar{j} + uv\,\bar{k}$ then $\bar{r}_u \cdot \bar{r}_u =$ ....

(7) The first form of Green's theorem is $\iiint\limits_R [f\bar{\nabla}^2 g + \bar{\nabla} f \cdot \bar{\nabla} g]dV =$ ....

(8) In triple integral ,total mass of density $\sigma(x, y, z)$ in region R is given by M = ...

Que.3 Attempt the following (Any Ten) [20]

(1) Evaluate $\int\limits_C 3(x^2 + y^2)ds$, where C : Over the path y = x from (0,0) to (1,1) (counterclockwise direction) .

(2) Evaluate $\int\limits_C [y^2 dx - x^2 dy]$, where C : along the straight line from (0,1) to (1,0).

(3) Evaluate $\int\limits_0^1 \int\limits_x^{2x} (1 + x^2 + y^2)\, dydx$.

(4) Find area of region R : $r = a(1 - \sin\theta)$.

(5) Define : Line integral independent of path , Exact differential equation .

(6) Change the order of integration in $\int\limits_0^c \int\limits_0^y f(x, y)\, dxdy$.

(7) Identify the surface $\bar{r} = a\cos v \cos u\bar{i} + a\cos v \sin u\bar{j} + a\sin v\bar{k}$.

(8) Represent the surface $\frac{x^2}{a^2} - \frac{y^2}{b^2} = z$ in parametric form .

(9) Represent the surface $x^2 + y^2 + z^2 = a^2$ in parametric form .

(10) Prove first form of Green 's theorem .

(11) In usual notation prove that $\iiint\limits_R \nabla^2 f\, dv = \iint\limits_S \frac{\partial f}{\partial n}\, dA$.

(12) Let R be a closed region in space and S be its boundary , let g be harmonic function in R then prove that $\iint\limits_S \frac{\partial g}{\partial n}\, dA = 0$.

Que.4 Attempt the following (Any Four) [32]

(1) Transform $\iint\limits_R (x^2 + y^2)\, dxdy$ in uv-plane by taking $x + y = u, x - y = v$.Then evaluate it, where R:Parallelogram with vertices (0,0),(1,1),(2,0),(1,-1).

(2) Let $f(x, y) = 1$ be the density of mass in region R : $0 \le y \le \sqrt{1 - x^2}$. $0 \le x \le 1$, then find centre of gravity and moments of inertia I_x, I_y, I_0 .

(3) Verify Green's theorem for $f = 3x^2 - 8y^2$, $g = 4y - 6xy$, C : the boundary of region bounded by $x = 0$, $y = 0$, $x + y = 1$.

(4) State and prove Green's theorem for plane .

(5) Evaluate $\iint_S f(x,y,z)\, dA$, where $f(x,y,z) = tan^{-1}(y/x)$,
S : $z = x^2 + y^2$; $1 \leq z \leq 4$, $x \geq 0$. $y \geq 0$.

(6) Find moment of inertia of surface S of density 1 about z-axis ,where
$S : \bar{r} = (a + bcosv)(cosu\ \bar{i} + sinu\ \bar{j}) + bsinv\ \bar{k}$, $a > b > 0$, $0 \leq u \leq 2\pi$, $0 \leq v \leq 2\pi$.

(7) State and prove Stoke's theorem.

(8) Verify Stoke's theorem for $\bar{V} = y^3\ \bar{i} - x^3\ \bar{j}$ and surface S : the circular disk $x^2 + y^2 \leq 1$, $z = 0$.

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