 No. of printed pages : 3

SARDAR PATEL UNIVERSITY
B.Sc. (III SEM.) (CBCS) EXAMINATION
2013
Wednesday, 2nd January
2.30 pm to 4.30 pm
US03EMTH05 : Calculus and Algebra-I

Total Marks: 70

Note: Figures to the right indicate full marks of the questions.

Q.1 Answer the following questions by selecting the most appropriate option. Write down the option in your answer book. [10]

(1) $\lim_{x \to 1} (4 - 4x^2)^{\frac{1}{\log(2-2x)}}$ is of the form ________.
(a) 1^∞ (b) ∞^0 (c) $\infty - \infty$ (d) 0^0

(2) $\lim_{x \to 1} (a - x)\tan\left(\frac{5\pi x}{2a}\right)$ = ________.
(a) 1 (b) $\frac{5\pi}{2a}$ (c) $\frac{2a}{5\pi}$ (d) 0

(3) $\lim_{x \to 2} \frac{\sin(x^2-4)}{(x-2)}$ = ________.
(a) 2 (b) 4 (c) 1 (d) 0

(4) Let z = f(x, y) be a real valued function defined on $E \subset R^2$. Suppose that f is homogeneous function of degree n. If f_x and f_y exists on E, then $x.\frac{\partial y}{\partial x} + y.\frac{\partial x}{\partial y}$ = ________.
(a) $n(n-1)z$ (b) $n.\frac{\phi(u)}{\phi'(u)}$
(c) nz (d) $(n-1)z$

(5) $f(x,y) = x^2y^4 - x^3y^3 + xy^5$ is homogeneous with degree n =________.
(a) 6 (b) 4 (c) 3 (d) 5

(6) A matrix is said to be skew-Hermitian if __________.
(a) $A^H = -A$ (b) $A^T = A$
(c) $A^H = A$ (d) $A^T = -A$

(7) The transpose of a matrix A is denoted by _________.
(a) A^* (b) $\bar{A}$ (c) A^T (d) A^{-1}

(8) A column matrix has only one _________.
(a) row (b) column (c) 1 (d) 0

(9) $(AB)^T$ = _________.
(a) AB (b) A^TB^T (c) B^TA^T (d) -AB

(10) If $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$ then $|A|$ = ________.
(a) 11 (b) 5 (c) -11 (d) -5

Q.2 Write down the answers of <u>ANY TEN</u> questions in short. [20]

(1) Evaluate $\lim_{x \to 0} \frac{\sin x - x + x^3}{x^3}$

(2) Evaluate $\lim_{x \to 0} \frac{x - \sin x}{\tan^3 x}$

(3) Evaluate $\lim_{x \to 0} \frac{e^x + \sin x - 1}{\log(1+x)}$

(4) Check whether the function $u = \frac{x^4 - xy^3}{x^3 + y^3}$ is homogeneous or not. If yes, find its degree.

(5) If $u = e^{ay} \cos ax$ then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

(6) For $u = x^3 - 3xy^2$, prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

(7) If $A = \begin{bmatrix} 2+5i & 7i \\ -3 & 1-i \end{bmatrix}$ then find A+A ^{H}

(8) Show that A-A^T is skew-symmertric.

(9) Define: Scalar Matrix with at least two illustrations.

(10) If $A = \begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix}$ then find characteristic equation for A.

(11) State distributive law for matrices.

(12) Define: Characteristic Matrix.

Q.3

(a) Find a, b, c so that $\lim_{x \to 0} \frac{ae^x - 2b\cos x + 3ce^{-x}}{x \sin x} = 2$ [05]

(b) Evaluate $\lim_{x \to 0} \frac{\log(\log(1-3x^2))}{\log(\log(\cos 2x))}$ [05]

OR

Q.3

(a) Evaluate $\lim_{x \to 0} \left(\frac{1}{x^2} - \cot^2 x\right)$ [05]

(b) Evaluate $\lim_{x \to 0} \left(\frac{\tan x}{x}\right)^{\frac{5}{3x^2}}$ [05]

Q.4

(a) Verify Euler's Theorem for $z = x^n \log\left(\frac{x}{y}\right)$ [05]

(b) If $u = \sin^{-1}\left(\frac{x^2 y^2}{x+y}\right)$, then prove that $x.\frac{\partial u}{\partial x} + y.\frac{\partial u}{\partial y} = 3\tan u$ [05]

OR

Q.4

(a) Find $x.\frac{\partial u}{\partial x} + y.\frac{\partial u}{\partial y}$ and $x^2.\frac{\partial^2 u}{\partial x^2} + 2xy.\frac{\partial^2 u}{\partial x dy} + y^2 \frac{\partial^2 u}{\partial y^2}$ for $u = \frac{x^2 y + xy^2}{x^2 + y^2}$ [05]

(b) If $u = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$ then prove that $x.\frac{\partial u}{\partial x} + y.\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = -u$ [05]

Q.5

(a) Show that for $K \in R$, [05]

(i) $K(A+\mathbf{A}^{\mathbf{H}})$ is Hermitian.

(ii) $K(A - \mathbf{A}^{\mathbf{H}})$ is skew-Hermitian.

(b) Define: Column Matrix, Unit Matrix, Skew-symmetric Matrix with illustration. [05]

OR

Q.5

(a) Show that every square matrix can be expressed as the sum of symmetric and skew-symmetric matrices. [05]

(b) Define: Row Matrix, Zero Matrix, Symmetric Matrix with illustration. [05]

Q.6 Verify Reversal law for [10]

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & 2 \\ 1 & -3 \end{bmatrix}$$

OR

Q.6 Verify Distributive law for [10]

$$A = \begin{bmatrix} 5 & 1 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix}, C = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$$

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