[30] No. of printed pages: 03

SARDAR PATEL UNIVERSITY

BSc. (III SEM.) (CBCS) EXAMINATION

Monday, 26th November 2012

10.30 am - 1.30 pm

US03CMTH01 : Mathematics (Advanced Calculus)

Total Marks: 70

Note: Figures to the right indicate full marks.

Q.1 Answer the following questions by selecting the most appropriate option. Write down the option in your answerbook. [10]

(1) If $\bar{r}(t)=a\cos t\,\bar{i}+b\sin t\,\bar{j}$; where a, b are constant, then $\bar{r}$ represents ______.

(a) Helix (b) Parabola

(c) Circle (d) Ellipse

(2) $\int_0^1 \int_x^{2x} dydx =$ ______.

(a) -3 (b) 3

(c) 1/2 (d) -1

(3) The line integral of a closed curve is ________.

(a) 0 (b) 1

(c) -1 (d) Does not exist

(4) Area of the regin R: r=a is ________.

(a) πa^2 (b) $\pi^2 a^2$

(c) $\pi^2 a$ (d) $a\pi$

(5) Area of plane region in polar form is given by A=______.

(a) $\int_C r^2 d\theta$ (b) $\frac{1}{2}\int_C r^2 d\theta$

(c) $\int_C r d\theta$ (d) $\frac{1}{2}\int_C r d\theta$

(6) $\oint_C \bar{u}\, d\bar{r}=0$ if and only if

(a) $\nabla\ \bar{u}=0$ (b) $\nabla\times\bar{u}=0$

(c) $\bar{u}$ is work less (d) none of these.

(7) Moment of Inertia of surface S about x-axis is denoted by ________.

(a) Iy (b) I_y

(c) Ix (d) I_x

(8) The unit normal vector to the surface f(x,y,z)=0 is ______.

(a) $\frac{\bar{\nabla} f}{|\bar{\nabla} f|}$ (b) $\frac{|\bar{\nabla} f|}{\bar{\nabla} f}$

(c) $\frac{\bar{\nabla} f}{|f|}$ (d) $\frac{|\bar{\nabla} f|}{f}$

(9) If f is harmonic function, then ________.

(a) $\nabla f = 0$ (b) $\nabla^2 f = 0$

(c) $\nabla^2 \vec{f} = 0$ (d) $\nabla \vec{f} = 0$

(10) The unit normal vector to the surface f(x,y,z) = 0 denoted by _______.

(a) n (b) N

(c) $\vec{n}$ (d) $\vec{N}$

Q.2 Write down any answer of **Any Ten** questions from the following in short. [20]

(1) Evaluate $\int_0^{\pi/2} \int_0^1 x^2 y^2 \, dxdy$.

(2) Discuss the work done by force.

(3) Write down the formula for moment of inertia about x-axis, y-axis and origin.

(4) Check whether the line integral $\int_{(2,0,0)}^{(1,2,3)} (xdx + ydy + zdz)$ is independent of path or not.

(5) State Green's theorem for plane.

(6) When a line integral is said to be independent of path?

(7) Describe area of surface $\bar{r}(u,\upsilon)$

(8) Evaluate $\iint_S \left[(x+z)dydz + (y+z)dzdx + (x+y)dxdy\right]$ where S: $x^2+y^2+z^2 = 1$.

(9) Write down the parametric form of a surface.

(10) State first form of Green's theorem.

(11) In usual notations prove that, $\iiint_R \nabla^2 f \, dv = \iint_S \frac{\partial f}{\partial n} dA$.

(12) State second form of Green's theorem.

Q.3

(a) Evaluate $\int_C 2xy^2 \, ds$ where C is a circle $x^2+y^2=1$ in xy-plane from a point (1,0) to (0,1). [05]

(b) Find the volume of the region bounded by the cylinder $x^2+y^2 = 4$ & y+z = 4, z=0. [05]

OR

Q.3

(a) Transform $\int_R \int (x^2 + y^2) dxdy$ in uv plane by taking x+y=u, x-y =v. Then evaluate it, where R: parallelogram with vertices (0,0), (1,1), (2,0) (1,-1). [05]

(b) Find volume of the region bounded by the first octant section cut from the region inside the cylinder $x^2+y^2=1$ and by the planes y=0, z=0, x=y. [05]

Q.4

(a) State and prove Green's theorem in vector form. [05]

(b) Find the area of the region in the first quadrant bounded by y=x, $y+x^3$. [05]

OR

Q.4

(a) State and prove Green's theorem for plane. [05]

(b) Find the area of the region R: r=a(1+cosθ). [05]

Q.5

(a) State and prove the first fundamental form of a surface in Cartesian form. [05]

(b) Evaluate $\int_s \int (2z(xy-x-y)dxdy + x^2\, dydz + y^2\, dzdx)$. [05]

OR

Q.5

(a) Evaluate $\int_s \int f(x,y,z)dA$ where $f(x,y,z) = \tan^{-1}(\frac{y}{x})$ [05]

S: $z = x^2+y^2$, $1 \le z \le 4$, $x \ge 0$, $y \ge 0$.

(b) By using divergence theorem, [05]

evaluate $\int_s \int (x^3 dydz + x^2 y\, dzdx + x^2 z\, dxdy)$ where S is closed surface bounded by the plane z=0, z=b, $x^2+y^2 = a^2$.

Q.6 State and prove Stoke's theorem. [10]

OR

Q.6 Verify Stoke's theorem for $\bar{v} = y^3 \bar{i} - x^3 \bar{j}$ and surface S: the circular disk $x^2+y^2 \le 1$, z=0.

****