(30|A-10)

SEAT No.________

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Sardar Patel University, Vallabh Vidyanagar

B.Sc. [Semester- VI] Examinations : 2018-19

Subject : Mathematics US06CMTH05 Max. Marks : 70

Graph Theory

Date: 03/04/2019, Wednesday Timing: 10.00 am - 01.00 pm

Instruction : The symbols used in the paper have their usual meaning, unless specified.

Q: 1. Answer the following by choosing correct answers from given choices. 10

[1] 20 members of a new club meet each day for meeting at a round table. They decide to sit such that every member has different neighbors at each meeting. How many days can this arrangement last?
[A] 9 [B] 10 [C] 18 [D] 20

[2] The maximum number of edges in a simple connected graph with 7 vertices is
[A] 7 [B] 14 [C] 21 [D] 28

[3] If degree of a vertex is zero then it is called
[A] a pendent vertex [B] an isolated vertex [C] circuit [D] path

[4] An operation of edge deletion on a graph removes corresponding
[A] edge only [B] vertices [C] vertices and edges both [D] none

[5] A Hamiltonian Path in a graph traverses through
[A] All vertices [B] All edges [C] All vertices and edges [D] none

[6] A conneccted graph with 7 vertices and 6 edges is
[A] a tree [B] a circuit [C] an Euler graph [D] none

[7] Rank of a connected graph is equal to the number of ____ in a spanning tree.
[A] all vertices [B] branches [C] chords [D] none

[8] If rank of a matrix is 9 and its nullity is 5 then the number of its edges is ___
[A] 4 [B] 5 [C] 9 [D] 14

[9] If r_1 and r_2 are ranks of two 1-isomorphic graphs G_1 and G_2 respectively then
[A] $r_1 < r_2$ [B] $r_1 > r_2$ [C] $r_1 = r_2$ [D] $r_1 = -r_2$

[10] A simple planar graph with 7 vertices and 10 edges has ____ faces
[A] 5 [B] 7 [C] 9 [D] 10

Q: 2. Answer any TEN of the following. 20

[1] Define : (i) Length of path (ii) Walk

[2] Define : (i) Edge disjoint subgraphs (ii) Open walk

P.T.O.

(1)

[3] Discuss Konigsberg bridge problem

[4] Define Euler Line and Euler Graph with an example.

[5] Define Complete graph with an example.

[6] Is graph of Königsberg bridge problem an Euler graph? Justify

[7] Define Spanning Tree, Branch and Chord with an example.

[8] Prove that the vertex connectivity of any graph G can never exceed the edge connectivity of G.

[9] Describe network flows

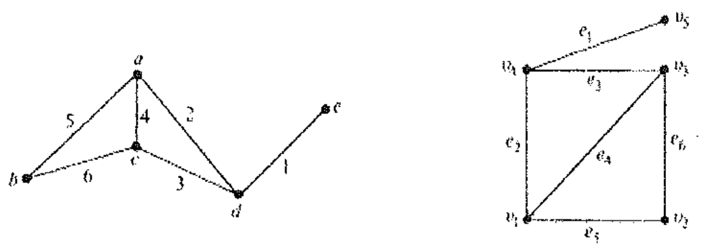
[10] Kurtowski's Second graphs

[11] For a simple connected planar graph with n-vertices , e-edges ($e > 2$) and f-regions prove the following.
(i) $e \geqslant \frac{3}{2}f$ (ii) $e \leqslant 3n - 6$

[12] Prove that a necessary condition for a graph G to be a planar graph is that G does not contain either of a Kuratowski's two graphs or any graph homeomorphic to either of them.

Q: 3 [A] Discuss (i) Utilities problem (ii) Seating problem 6

[B] Explain Isomorphism between two graphs and examine whether following pairs of graphs are isomorphic or not.



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OR

Q: 3 [A] Prove that a graph G is disconnected *iff* its vertex set V can be partitioned into two non-empty disjoint subsets V_1 and V_2 such that there exists no edge in G whose one end vertex is in subset V_1 and other in subset V_2 5

[B] If a graph (connected or disconnected) has exactly two vertices of odd degree then prove that there must be a path joining these two vertices. 5

Q: 4 [A] Prove that every connected graph G is an Euler graph *iff* it can be decomposed into circuits. 5

[B] Prove that there is one and only one path between every pair of vertices in a tree. **5**

OR

Q: 4 [A] Prove that a graph G with n-vertices and $n-1$ edges and no circuits is connected. **5**

[B] Prove that a graph is a tree *iff* it is minimally connected. **5**

Q: 5 [A] Describe a method to find all spanning tree of a graph. **5**

[B] Prove that every cut-set in a connected graph G must contain atleast one branch of every spanning tree. **5**

OR

Q: 5 [A] Prove that with respect to a given spanning tree T, a chord c_i that determines fundamental circuit τ, occurs in every fundamental cut-sets associated with the branches in τ and in no other cut-sets. **5**

[B] Prove that a vertex v in a connected graph G is a cut-vertex iff there exist two vertices x and y in G such that every path between x and y passes through v. **5**

Q: 6 [A] Define Circuit correspondence and prove that 2-isomorphic graphs have circuit correspondence. **5**

[B] Using Euler's theorem prove that Kuratowski's first and second graphs are non-planar. **5**

OR

Q: 6 [A] Give an example to show that dual of dual of a graph may not be isomorphic to the original graph. **5**

[B] Define Geometric dual and find geometric dual of the following graph 5