[59/A-14]

SEAT No.________ No. of Printed Pages : 2

SARDAR PATEL UNIVERSITY

B.Sc.Sem-6 EXAMINATION

1^{st} April, 2019, Monday

10:00 a.m. to 01:00 p.m.

US06CMTH04(MATHEMATICS)

(Abstract algebra-II)

Maximum Marks: 70

Q.1 Choose the correct option in the following questions, mention the correct option in the answer book. [10]

(1) .... is a Skew field but not a field .
(a) Ring of real quaternion (b) $\mathbb{Q}$ (c) $M_2(\mathbb{R})$ (d) $\mathbb{Z}$

(2) .... is regular element of $\{a + b\sqrt{-5}/a, b \in \mathbb{Z}\}$.
(a) 0 (b) $\{\pm i\}$ (c) $\{\pm 1\}$ (d) $\{1 + \sqrt{-5}\}$

(3) Characteristic of every field is either zero or ....
(a) prime (b) 4 (c) not prime (d) integer

(4) Quotient field of ring of Gaussian integer is ....
(a) $\mathbb{Z}$ (b) $\mathbb{Q}$ (c) $\mathbb{Z} + i\,\mathbb{Z}$ (d) $\mathbb{Q} + i\,\mathbb{Q}$

(5) .... is maximal ideal of field .
(a) 0 (b) $\{1\}$ (c) $\{0\}$ (d) none of these

(6) If I is ideal in ring R then unit element of R/I is ....
(a) 0 (b) 1 (c) R (d) $1 + I$

(7) In $\mathbb{Z} + i\mathbb{Z}$, gcd of 2 and $-1 + 5i$ is ....
(a) $2 + i$ (b) $2 - i$ (c) i (d) $1 - i$

(8) If $n \in \mathbb{Z}$, $n > 1$ is irreducible then n is ....
(a) 4 (b) 0 (c) prime (d) not prime

(9) If $R = Z + iZ$, $f(x) = 2x^2 - (1+i)x - 2$ then content of f is ....
(a) $2 + i$ (b) $2 - i$ (c) $1 - i$ (d) $1 + i$

(10) If R is an integral domain , $f(x), g(x) \in R[x]$ then degree(fg) .... degree f + degree g.
(a) $>$ (b) $\leq$ (c) $=$ (d) $\geq$

Q.2 Attempt the following (Any ten). [20]

(1) Prove that ring $C[0,1]$ is ring with zero divisor under point wise multiplication.
(2) Find Characteristic of ring Z_5.
(3) Define Root of polynomial and Content of polynomial.
(4) Let $f : R \to R'$ be ring homomorphism ,then prove that Kerf is an ideal in R.
(5) Find Z_6/I , *where* $I = \{\bar{0}, \bar{2}, \bar{4}\}$.
(6) Let $R = C[0,1]$.Prove that $I = \{\ x\ /x \in R,\ x(1/2) = 0\}$ is an ideal in R.
(7) Define Euclidean Domain with example.
(8) Find all the regular elements of Z_n.
(9) Find gcd of 2+3i and 4+7i in Z+iZ.
(10) If p is prime then prove that $x^n - p \in Z[x]$ is irreducible.
(11) Find content of f where $f(x) = 3x^3 - 2x^2 + 6x + 9 \in Z[x]$.
(12) Show that 1+i is irreducible in Z+iZ.

(P.T.O)

Q.3

(a) State and Prove Cayley's theorem for rings. [5]

(b) Prove that the characteristic of a field is either 0 or a prime. [5]

OR

Q.3

(c) Let $R = \mathbb{C}$, $R' = \left\{ \begin{pmatrix} a & -b \\ b & a \end{pmatrix} / a, b \in \mathbb{R} \right\}$,then prove that $R \simeq R'$. [5]

(d) Prove that the only isomorphism of $\mathbb{R}$ onto $\mathbb{R}$ is the identity map $I_{\mathbb{R}}$. [5]

Q.4

(a) If R is a commutative ring with 1, then prove that every maximal ideal in R is a prime ideal. [5]

(b) State and Prove First isomorphism theorem for ring. [5]

OR

Q.4

(c) Let R be a commutative ring with 1.Then prove that an ideal M is a maximal ideal iff R/M is a field. [5]

(d) Prove that every simple ring need not be a field. [5]

Q.5

(a) Show that the ring of Gaussian integers is Euclidean domain. [5]

(b) Let R be a principal ideal domain ,then prove that every $a \in R$ which is not a unit can be expressed as a product of irreducible elements. [5]

OR

Q.5

(c) Prove that every principal ideal domain is factorization domain. [5]

(d) Show that $1 + 2\sqrt{-5}$ is an irreducible element but not a prime element in $\{a + b\sqrt{-5} / a, b \in Z\}$. [5]

Q.6

(a) State and Prove Gauss lemma. [5]

(b) Let F be a field and $f(x) \in F[x]$ be of degree 2 or 3.Then prove that f(x) is reducible iff f(x) has a root in F. [5]

OR

Q.6

(c) Prove that every UFD need not be a PID . [5]

(d) State and Prove Eisenstein's criterion. [5]

—X—