[82/A-16] **SEAT No.**______ **No. of Printed Pages : 3**

Sardar Patel University, Vallabh Vidyanagar

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Subject : Mathematics US06CMTH03 Max. Marks : 70

Topology

Date: 29/03/2019, Friday Timing: 10.00 am - 01.00 pm

Instruction : The symbols used in the paper have their usual meaning, unless specified.

Q: 1. Answer the following by choosing correct answers from given choices. **10**

[1] In $(\mathcal{R},\mathcal{U})$, the set $[a,b]-\{\frac{a+b}{2}\}$ is
[A] Open [B] Closed [C] Open as well closed [D] neither open nor closed

[2] In a topological space $(X,\mathcal{T})$, a neighbourhood of a point is
[A] $\mathcal{T}$-open [B] $\mathcal{T}$-closed [C] either open or closed [D] none

[3] In indiscrete topology for a non-empty set X with more than one elements, every subset of X is
[A] open only [B] closed only [C] open and closed both [D] none

[4] In $(R,\mathcal{U})$, the set $\{1,2,3,4\}$ has ____ cluster point/s.
[A] no [B] one [C] two [D] four

[5] In $(\mathcal{R},\mathcal{U})$ which of the following is not dense
[A] $\mathcal{R}$ [B] $\mathcal{Q}$ [C] $\mathcal{J}^+$ [D] $\mathcal{R}-\mathcal{Q}$

[6] In $(\mathcal{R},\mathcal{U})$ which of the following is not closed
[A] $\emptyset$ [B] $\mathcal{R}$ [C] $(1,2)$ [D] $[1,2]$

[7] In its relativised topology, the subset ____ of R is connected.
[A] $[0,5)$ [B] $[0,4]-\{1\}$ [C] $(0,1)\cup(1,2)$ [D] none

[8] Every non-empty and bounded below subset of R possesses
[A] the g.l.b. in R [B] the l.u.b. in R
[C] g.l.b. and l.u.b. in R [D] none

[9] If every open cover of a topological space has a finite subcover then it is
[A] Compact [B] Unbounded [C] a Regular Space [D] none

[10] If (X,T) is a T_2 space and $a\in X$ then $\{a\}$ is
[A] open [B] closed [C] closed and open both [D] neither open not closed

Q: 2. Answer ANY TEN of the following. **20**

[1] Define : (i) Topology (ii) Coarser Topology

[2] What are trivial topologies on a non-empty set?

(P.T.O.)

[3] If $X = \{a, b, c\}$ then find three topologies $\mathcal{T}_1$, $\mathcal{T}_2$ and $\mathcal{T}_3$ for X such that $\mathcal{T}_1 \subsetneqq \mathcal{T}_2 \subsetneqq \mathcal{T}_3$

[4] Find $\mathcal{U}$-closures of the sets $\mathbb{R}$ and $\emptyset$.

[5] Let $(X, \mathcal{T})$ be a topological space. Prove that if F is $\mathcal{T}$-closed subset of X and $p \in (X \sim F)$ then there is a $\mathcal{T}$-neighbourhood N of p such that $N \cap F = \emptyset$

[6] For any topologies $\mathcal{T}$ and Ψ of $\mathbb{R}$ show that the mapping $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = 2, \forall x \in \mathbb{R}$, is $\mathcal{T}$-Ψ continuous

[7] For $X = \{0, 1, 2, 3, 4, 5\}$ consider the topology $\mathcal{T} = \{X, \emptyset, \{0, 1, 2\}, \{3, 4, 5\}\}$. Is $(X, \mathcal{T})$ connected?

[8] Prove that indiscrete space is connected

[9] Let $f : [0, 1] \to R$ be continuous on $[0, 1]$ and be onto R also. Is $f([0, 1])$ connected?

[10] Prove that the space $(R, \mathcal{U})$ is a T_2-space.

[11] Give an example of a T_1-space that is not a T_2-space

[12] Define : (i) Regular Space (ii) Bounded Mapping

Q: 3 [A] Define Closed Set. Also if $(X, \mathcal{T})$ is a topological space and $\{F_\alpha / \alpha \in \Lambda\}$ is any collection of $\mathcal{T}$-closed subsets of X then prove that $\bigcap\{F_\alpha / \alpha \in \Lambda\}$ is a $\mathcal{T}$-closed set — 5

[B] Consider the topology $\mathcal{G}$ on $\mathbb{R}$ where $G \subset \mathbb{R}$ is $\mathcal{G}$-open if $G = \emptyset$ or $G \neq \emptyset$ and for each $p \in G$ there is a set $H = \{x \in \mathbb{R} / a \leqslant x < b\}$ for some $a < b$ such that $p \in H \subset G$. Prove that $\mathcal{G}$ is finer than usual topology of $\mathbb{R}$ — 5

OR

Q: 3 [A] Let $(X, \mathcal{T})$ be a topological space and let A be a subset of X. Prove that A is $\mathcal{T}$-open set iff A contains a $\mathcal{T}$-neighbourhood of each of its points — 5

[B] Let J be the set of all integers and $\mathcal{J}$ be a collection of subsets G of J where $G \in \mathcal{J}$ whenever $G = \emptyset$ or $G \neq \emptyset$ and $p, p \pm 2, p \pm 4, ..., p \pm 2n, ...$ belong to G whenever $p \in G$. Prove that $\mathcal{J}$ is a topology for J — 5

Q: 4 [A] Let $(X, \mathcal{T})$ be a topological space and let A be a subset of X. Then prove that $A^- = A \cup A'$. — 5

[B] Let $(X, \mathcal{T})$ be a topological space and let A be a subset of X and A' be the set of all cluster points of A. Prove that A is $\mathcal{T}$-closed iff $A' \subset A$ — 5

OR

Q: 4. If $(X, \mathcal{T})$ and (Y, Ψ) are topological spaces and f is a mapping from X into Y then prove that the following statements are equivalent

(a) The mapping f is continuous

(b) The inverse image of f of every Ψ-closed set is $\mathcal{T}$-closed set

(c) If $x \in X$ then inverse image of every Ψ-neighbourhood of $f(x)$ is a $\mathcal{T}$-neighbourhood of x

(d) If $x \in X$ and N is a Ψ-neighbourhood of (x), then there is a $\mathcal{T}$-neighbourhood M of x such that $f(M) \subset N$

(e) If $A \subset X$, then $f(A^-) \subset f(A)^-$ **10**

Q: 5. Define a connected space and prove that the space $(R, \mathcal{U})$ is connected. **10**

OR

Q: 5 [A] Prove that if $(X, \mathcal{T})$ is disconnected then there is a nonempty proper subset of X that is both $\mathcal{T}$-open and $\mathcal{T}$-closed. **5**

[B] If $(X, \mathcal{T})$ is compact and Y is a $\mathcal{T}$-closed subset of X, then prove that $(Y, \mathcal{T}_Y)$ is also compact. **5**

Q: 6 [A] If $Y \subset R$ and the space $(Y, \mathcal{U}_Y)$ is compact, then prove that Y is bounded and $\mathcal{U}$-closed. **5**

[B] Let $(X, \mathcal{T})$ and (Y, ψ) be topological spaces, and let f be a $\mathcal{T} - \psi$ continuous mapping of X onto Y. If $(X, \mathcal{T})$ is compact then prove that (Y, ψ) is also compact. **5**

OR

Q: 6 [A] Prove that every compact Hausdorff space is a T_3-space. **5**

[B] If $(X, \mathcal{T})$ is a compact space, and if (Y, ψ) is a Hausdorff space, and if f is a one-to-one $\mathcal{T} - \psi$ continuous mapping of X onto Y, then prove that f is a homeomorphism. **5**

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